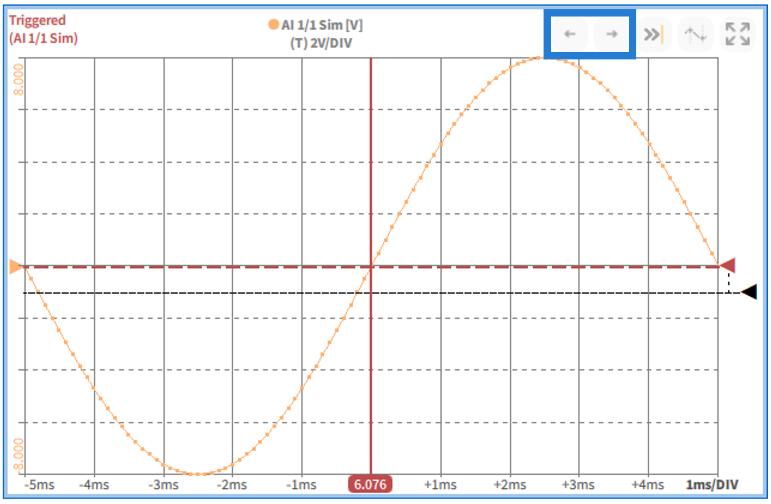


Fig. 8.39: Scope in PLAY mode with arrow buttons

8.12 Spectrum analyzer

The FFT-Instrument provides the user with the ability to analyze data in real time within the frequency domain.

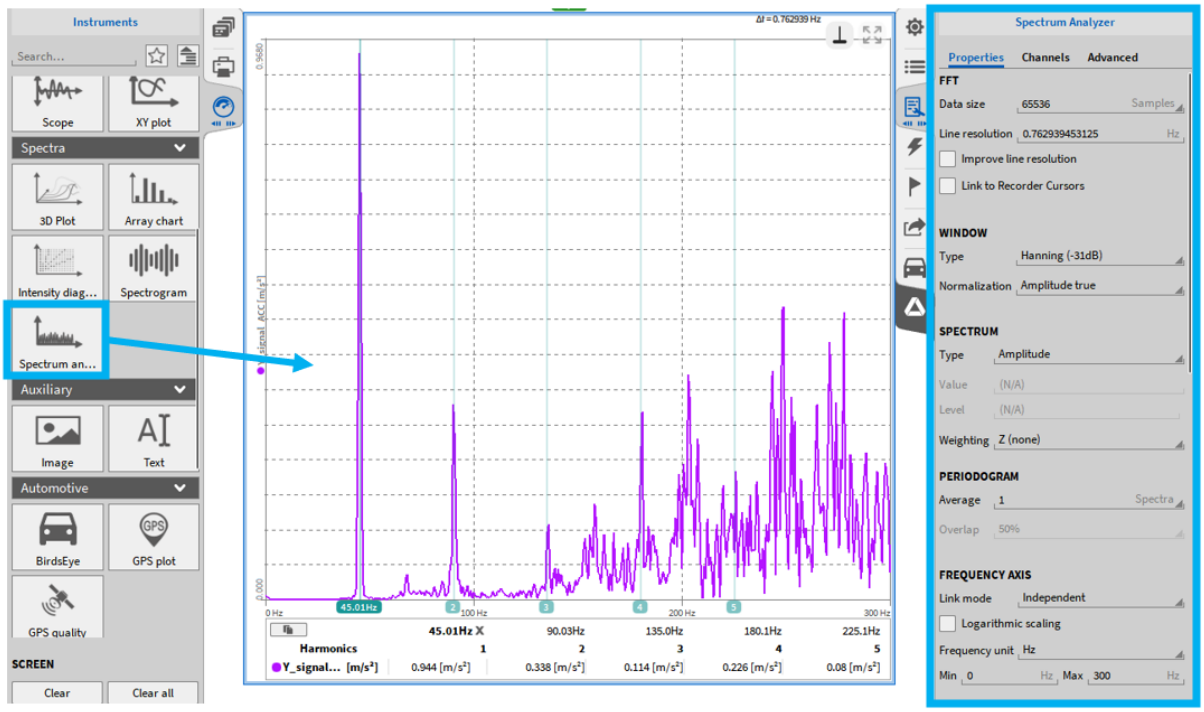


Fig. 8.40: Spectrum Analyzer - overview

The main properties of the instrument are FFT, Window, Spectrum, Periodogram, Frequency Axis, Value Axis, Markers, Reference curve, Style and Cross Hairs.

Both time domain and frequency domain channels can be added to the spectrum analyzer. Frequency domain channels are for example the amplitude channel created by a FFT channel from the basic math options.

8.12.1 Assignment of Frequency Domain Channels

Mathematical frequency channels that are calculated using the FFT math (see FFT channels) can be assigned and displayed to the Spectrum Analyzer as well. The Amplitude channel (called Channel_Name_Amp per default) and the Phase channel (called Channel_Name_Phi per default) can be assigned to the Spectrum Analyzer but no complex FFT channels (called Channel_Name_Cpx per default).

Note:

- Time domain channels and frequency domain channels cannot be assigned to the same Spectrum Analyzer but only to separate ones.
- If frequency domain channels are assigned to the Spectrum Analyzer, the Instrument Properties are reduced to the Frequency axis and Value Axis settings (see Fig. 8.41). For details, refer to Additional instrument properties.

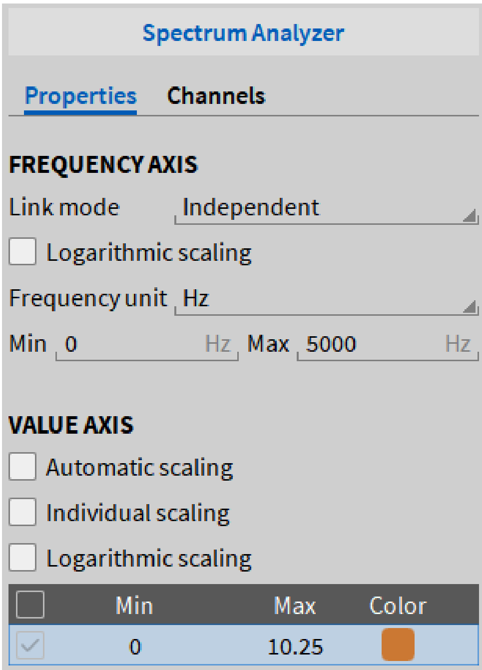


Fig. 8.41: Instrument Properties of the Spectrum Analyzer if Frequency Domain channels are assigned

8.12.2 Frequency Axis Settings

The unit of the X-axis is Hertz [Hz] per default (see ② in Fig. 8.42). The unit can be changed to Cycles Per Minute [CPM] which is defined as [Hz] * 60. The axis' minimum can be freely defined (see ③ and ④ in Fig. 8.42). The scaling can optionally bet set from linear to logarithmic scaling (see ① in Fig. 8.42).

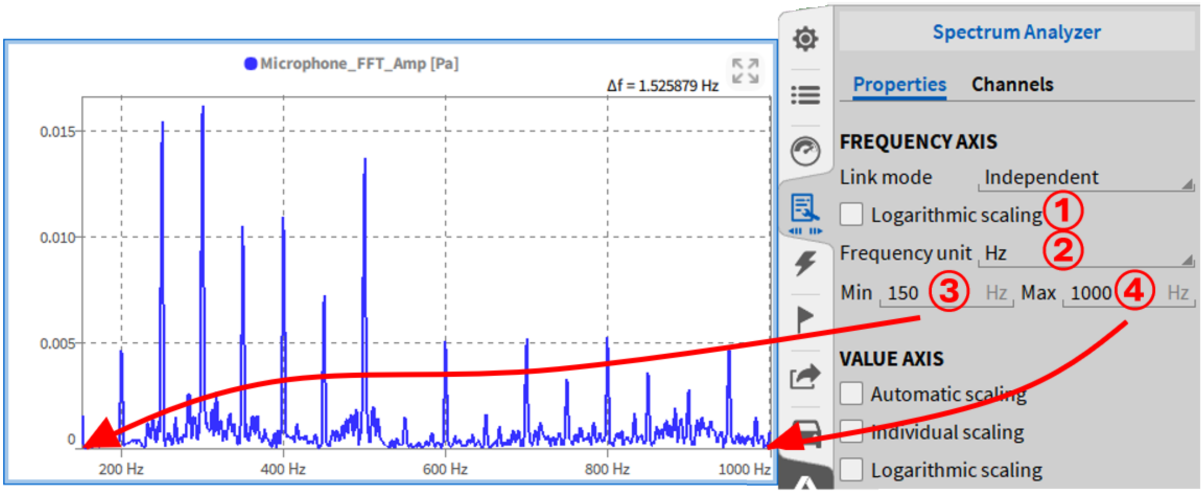


Fig. 8.42: Frequency axis settings

8.12.3 Assignment of Time Domain Channels

If analog channels that represent a time domain signal are assigned to the Instrument, the FFT is calculated according to the following formula:

$$Y_k = \sum_{n=0}^{N-1} X_n e^{\frac{-i2\pi kn}{N}}; \qquad k = 0 \ldots N-1$$

Xk… (complex) input signal

Yk… complex Fourier Transform of Xk

N… number of samples

Depending on the spectrum to be plotted, the complex Fourier Transform Y_k is used for further calculations. For continuative information, refer to *Section Spectrum*.

Note:

- Up to 8 channels can be assigned to one single Spectrum analyzer.
- The Spectrum analyzer provides the zooming option as well. For the detailed description of the zooming function, refer to *Pinch/Scroll zoom feature*.
- The user can easily export the currently displayed FFT-spectrum via pressing **CTRL+C** and paste it into an Excel file or Notepad window
- Peak Hold function: To facilitate the read off from local maxima, the user can press the **SHIFT** key. This makes the cursor remain at local maxima.

8.12.4 FFT properties for Time Domain Channels

The desired *Data size* (i.e. the number of samples in time domain used for the calculation of one spectrum which is denoted with *N* in the upper formula) can be edited here. The data size is freely definable within a range from 42 to 16777216 (2^{24}) samples. The default settings are

1024 (2^{10}), 2048 (2^{11}), 4096 (2^{12}), 8192 (2^{13}), 16384 (2^{14}), 32768 (2^{15}), 65536 (2^{16}) 131072 (2^{17}), 262144 (2^{18}), 1048576 (2^{20}), 4194304 (2^{22}) and 16777216 (2^{24}) samples..

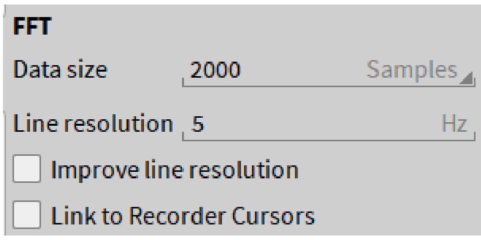


Fig. 8.43: FFT property of the spectrum analyzer instrument

The Line resolution relates to the sample rate and the Data size:

$$Line\ Resolution = \frac{\text{Samplerate}}{\text{Window size}}\ [Hz]$$

The radio button *Improve line resolution* will enable zero-padding. For detailed information, refer to *Additional information: improve line resolution (Enable zero-padding)*.

Note:

- If channels with different sample rates are displayed in one Spectrum analyzer:
- The *Line resolution* is calculated for each sample rate individually and cannot be edited in the Instrument Properties. Thereby, the number of plotted FFT bins is the same for each signal but the FFT resolution is different.
- Zero-padding (*Improve line resolution*) cannot be activated.
- Note that changing the *Data size* will affect the *Line resolution*. Therefore, the line resolution is within a range from $\frac{\text{Samplerate}}{2^{20}}$ to $\frac{\text{Samplerate}}{42}$ samples.
- If *Improve line resolution* is de-selected, the number of calculated FFT bins is equal to the *Data size*. If *Improve line resolution* is selected, the number of calculated FFT bins is always higher than the number of data samples.
- The number of plotted FFT bins is always $trunc(\frac{\text{Number of calculated frequency bins}}{2}) + 1$. The first line is plotted @ 0 Hz and the last line is plotted @ $\frac{\text{Samplerate}}{2}$ Hz. If logarithmic frequency axis scaling is selected, the 0 Hz line will not be plotted, because the common logarithm is not defined for 0.

Section *Window*

The *Type* and *Normalization* of the window function can be edited here.

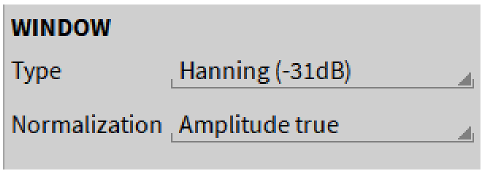


Fig. 8.44: Window settings for the spectrum analyzer

Window type

The Spectrum analyzer offers the usage of 7 different window functions (N denotes the Window size in samples and corresponds to the *Data size*):

- Hanning window

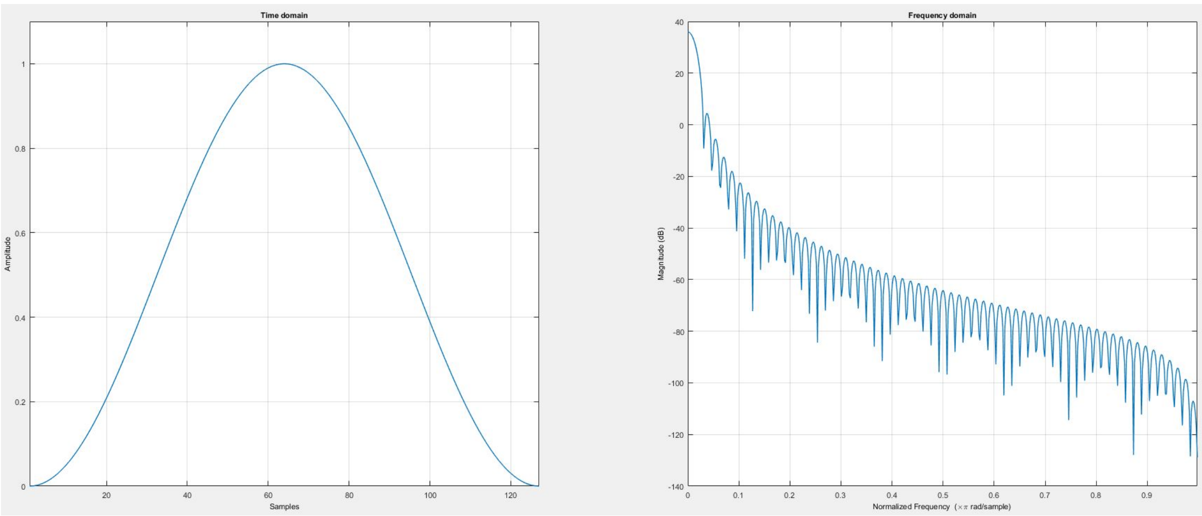


Fig. 8.45: Hanning window in time and frequency domain (N = 128)

$$w(n) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{N-1}\right)\right]; \qquad n = 0 \dots N-1$$

- Hamming window

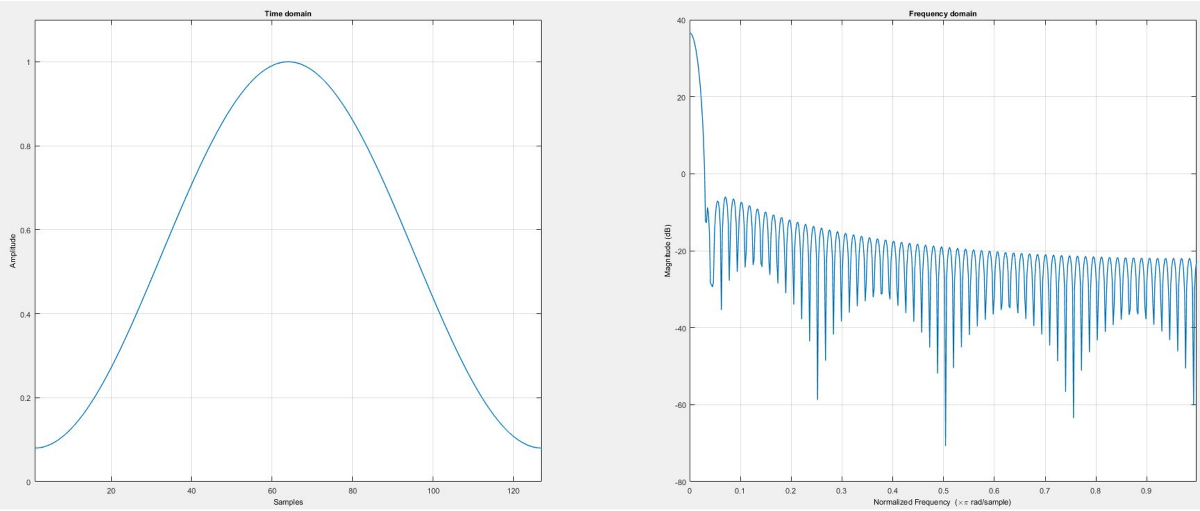


Fig. 8.46: Hamming window in time and frequency domain (N = 128)

$$w(n) =\ \alpha - \beta \cos\left(\frac{2\pi n}{N-1}\right); \qquad n = 0 \ldots N-1$$

α = 0.54

β… *1 – α*

- Rectangular window

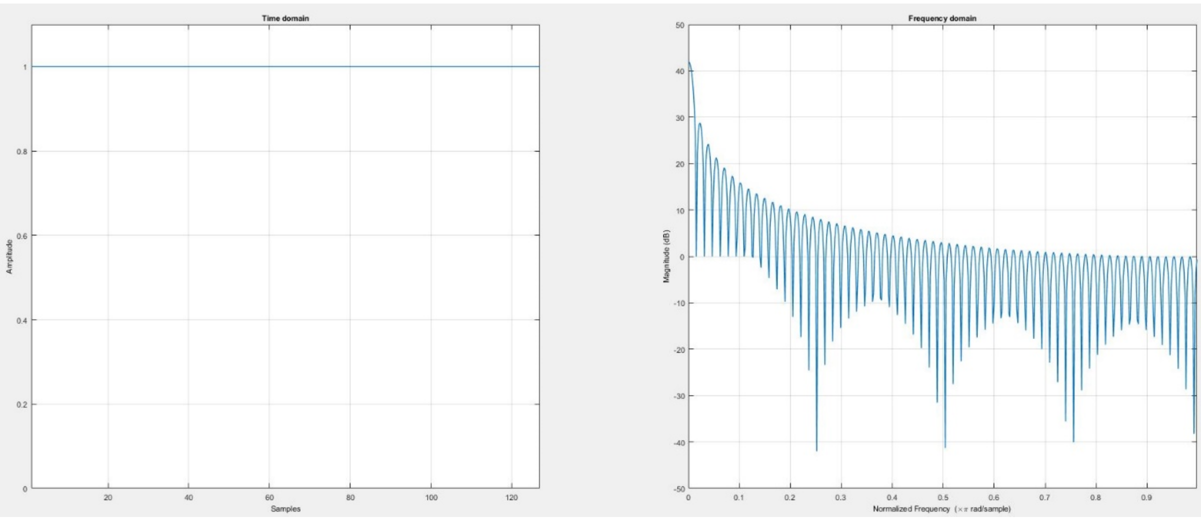


Fig. 8.47: Rectangular window in time and frequency domain (N = 128)

$$`w(n) =\ 1\,; \qquad n = 0 \ldots N` - 1$$

- Blackman window

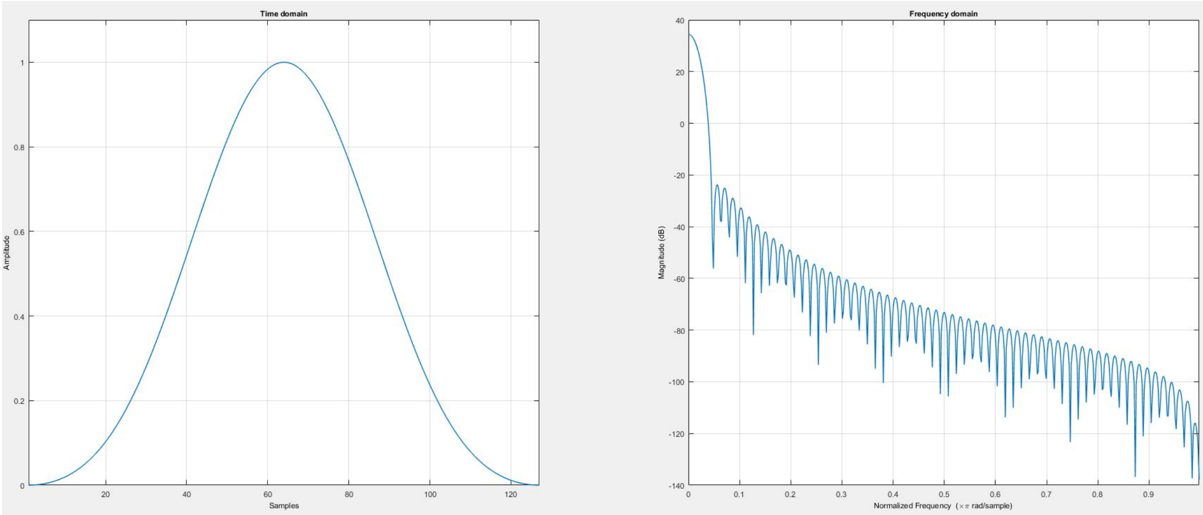


Fig. 8.48: Blackman window in time and frequency domain (N = 128)

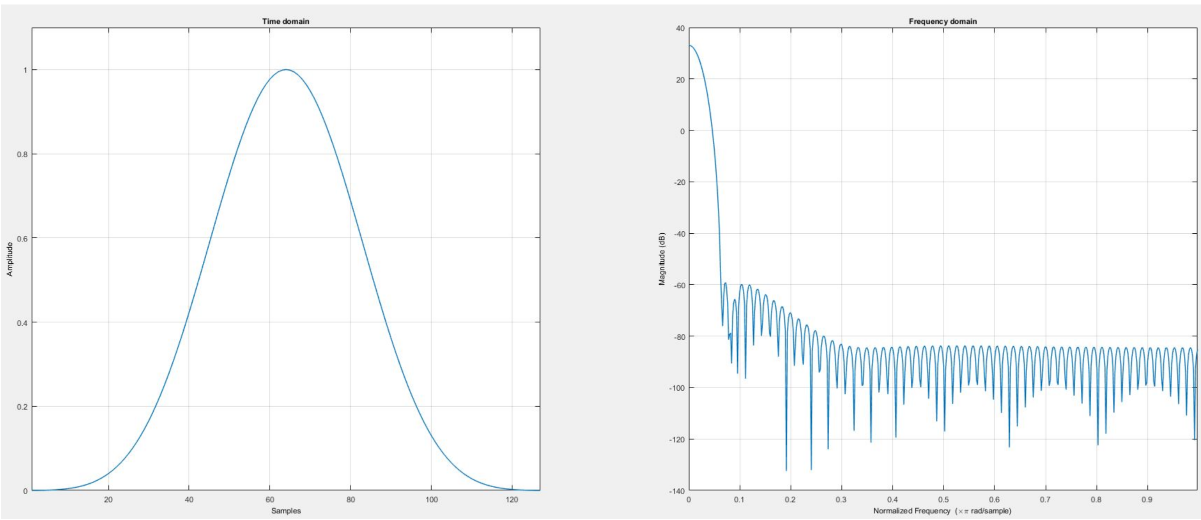
$$w(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right) + a_2 \cos\left(\frac{4\pi n}{N-1}\right); \quad n = 0 \ldots N-1$$

a0 = 0.42

a1 = 0.5

a3 = 0.08

- Blackman-Harris window



Blackman-Harris window in time and frequency domain (N = 128)

$$w(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right) + a_2 \cos\left(\frac{4\pi n}{N-1}\right) - a_3 \cos\left(\frac{6\pi n}{N-1}\right); \quad n = 0 \ldots N-1$$

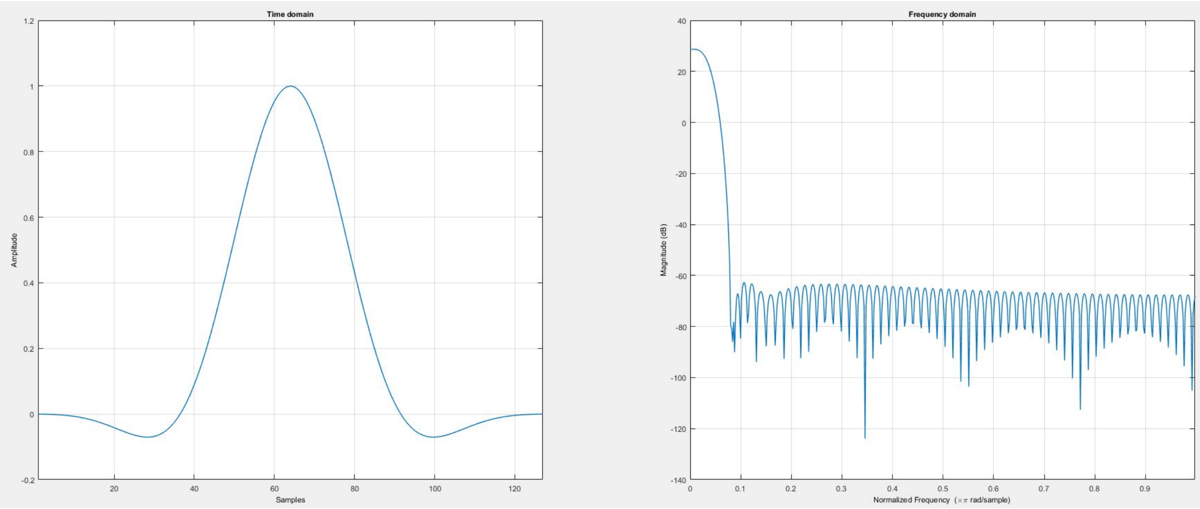
a0 = 0.35875

a1 = 0.48829

a2 = 0.14128

a3 = 0.01168

- Flat-Top window



Flat-Top window in time and frequency domain (N = 128)

$$w(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right) + a_2 \cos\left(\frac{4\pi n}{N-1}\right) - a_3 \cos\left(\frac{6\pi n}{N-1}\right) + a_4 \cos\left(\frac{8\pi n}{N-1}\right); n = 0 \ldots N-1$$

a0 = 0.21557895

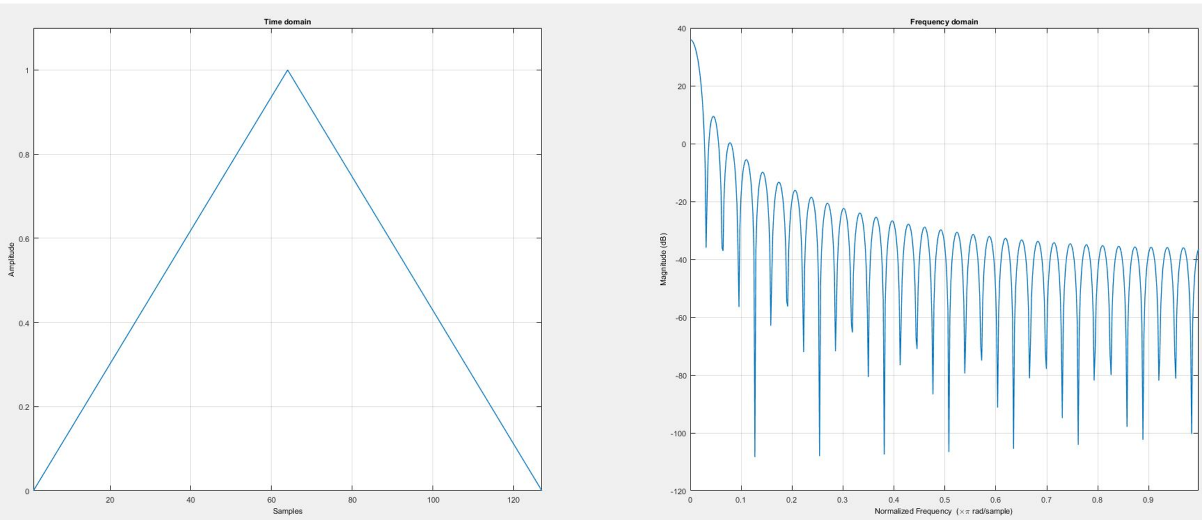
a1 = 0.41663158

a2 = 0.277263158

a3 = 0.083578947

a4 = 0.006947368

- Bartlett window



Bartlett window in time and frequency domain (N = 128)

$$w(n) = 1 - \left|\frac{n - \frac{N-1}{2}}{\frac{N-1}{2}}\right|$$

The following table will give an overview and recommendations about the usage of the different window functions.

Note: This table is only a matter of recommendation and makes no claim to be complete or correct.

Table 8.2: Recommendation about the usage of different window functions (Source)

Signal Content	Window
Sine wave or combination of sine waves	Hanning
Sine wave (amplitude accuracy is important)	Flat Top
Narrow-band random signal (vibration data)	Hanning
Broadband random (white noise)	Rectangular
Closely spaced sine waves	Rectangular, Hamming
Unknown Content	Hanning
Accurate single tone amplitude measurements	Flat Top

The following figure compares the different window functions in time domain:

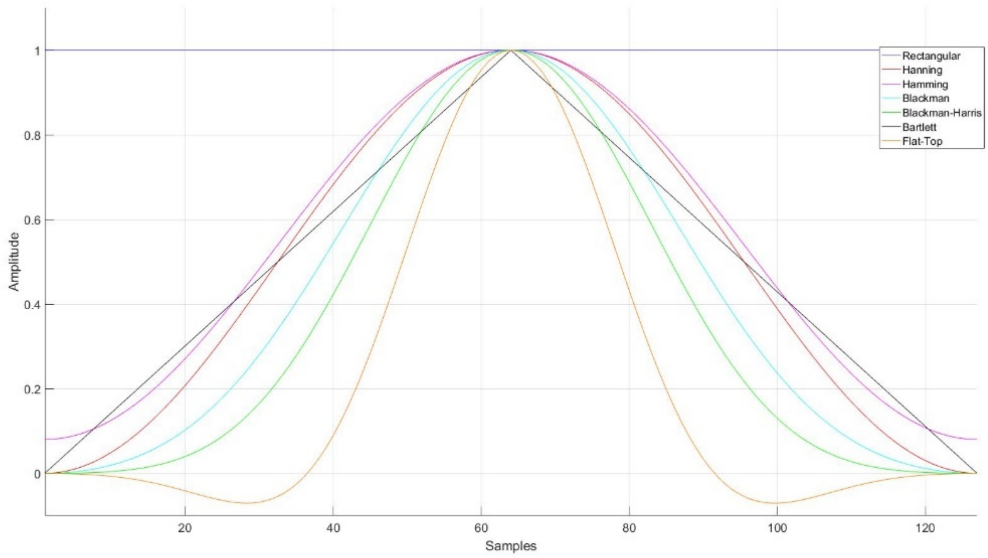


Fig. 8.49: Comparison of the window functions in time domain (N = 128)

The following table summarizes the two most important characteristics of the different window functions. The *Main Maximum Width* describes the single-sided width of the main maximum as number of FFT bins. The *Main Maximum Width* in Hz is the product of *Main Maximum Width* and *Line resolution*. The *Max. Side Lobe Level* denotes the damping of the first side lobe compared to the main maximum in decibel.

Table 8.3: Properties of the window functions

Window function	Main Maximum Width	Max. Side Lobe Level [dB]
Hanning	2	-31
Hamming	2	-43
Rectangular	1	-13
Blackman	3	-58
Blackman-Harris	4	-92
Flat-Top	5	-68
Bartlett	2	-27

Normalization

As the usage of a window function causes a decrement of the signals' amplitude and power, the user can select between *None*, *Amplitude True* and *Power True* Normalization.

- *None:* The spectrum will not be normalized, and the amplitude and the power error will remain
- *Amplitude True:* The damping of the signal amplitude caused by the window function will be compensated. The power loss will remain. The correction happens according to the following formula:

$$S_{\text{AmpCorr}\ k} = S_k * \left[\frac{N}{\sum_{k=1}^{N} W_k} \right]$$

- *Power True:* The Power loss caused by the multiplication with the window function will be compensated and the amplitude error will remain. The correction happens according to the following formula:

$$S_{\text{PowCorr}\ k} = S_k * \sqrt{\frac{N}{\sum_{k=1}^{N} W_k^2}}$$

  Sk… Un-normalized signal at position k

  N… Length of the Window function

  Wk…Value of the window function at position k

- A detailed example for the necessity to normalize FFT spectra can be found in *Normalization of FFT Spectra*.

Note: The normalization is applied to the signal in time domain.

Section *Spectrum*

In the *Spectrum* section, the user can select the type of spectrum plotted in the Spectrum analyzer. In the following section, the available spectra and their formula are listed.

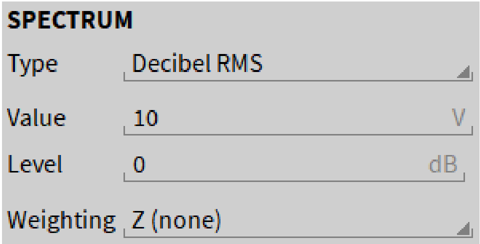


Fig. 8.50: Spectrum settings for the Spectrum Analyzer

- Amplitude: Plots the default amplitude spectrum normalized to the number of FFT lines according to the following formula:

$$A_k = \frac{1}{N}\sqrt{\text{Re}\left\{Y_k\right\}^2 + \text{Im}\left\{Y_k\right\}^2}\,; \qquad k = 0 \qquad [\text{Unit}]$$

$$A_k = \frac{2}{N}\sqrt{\text{Re}\left\{Y_k\right\}^2 + \text{Im}\left\{Y_k\right\}^2}\,; \qquad k = 1\ldots N \qquad [\text{Unit}]$$

- Amplitude RMS: Plots the RMS amplitude spectrum by dividing the Amplitude spectrum by .

$$A_{\text{RMS}\ k} = \frac{A_k}{\sqrt{2}}; \qquad k = 1\ldots N \qquad [\text{Unit}]$$

- Amplitude²: Plots the squared amplitude spectrum by squaring the Amplitude spectrum

$$A_{\text{sq}\ k} = A_k^2; \qquad k = 1\ldots N \qquad [(\text{Unit})^2]$$

- Amplitude P2P: Plots Peak-2-Peak amplitude spectrum which is the amplitude spectrum normalized to the number of FFT lines multiplied by 2 according to the following formula:

$$A_k = \frac{4}{N}\sqrt{\text{Re}\left\{Y_k\right\}^2 + \text{Im}\left\{Y_k\right\}^2}\,; \qquad k = 1\ldots N \qquad [\text{Unit}]$$

  For k = 0 the Peak-2-Peak amplitude spectrum is 0 per definition.

- Decibel: Plots the logarithmic Amplitude spectrum referred to a freely definable reference level A_{Ref}. The reference value A_{ref} can be edited in the Value section and its corresponding level can be defined in the Level section.

$$L_{A\ k} = 20 * \log_{10}\left(\frac{A_k}{A_{\text{Ref}}}\right); \qquad k = 1\ldots N \qquad [\text{dB}]$$

- Decibel RMS: Plots the logarithmic Amplitude RMS spectrum referred to a freely definable reference level A_{Ref}. The reference value A_{ref} can be edited in the Value section and its corresponding level can be defined in the Level section.

$$L_{A\ \text{RMS}\ k} = 20 * \log_{10}\left(\frac{A_{\text{RMS}\ k}}{A_{\text{Ref}}}\right); \qquad k = 1\ldots N \qquad [\text{dB}]$$

- Decibel Max Peak: Plots the logarithmic Amplitude spectrum referred to the highest occurring value in the Amplitude spectrum. Thus, the highest occurring value corresponds to 0 dB.

$$L_{A\ \text{Max}\ k} = 20 * \text{log}_{10}\left(\frac{A_k}{\text{max}\{A_k\}}\right); \qquad k = 1 \ldots N \quad [\text{dB}]$$

- Decibel V-RMS: Plots the logarithmic Amplitude spectrum referred to 1 [Signal Unit] (1 V (RMS) is a common reference level for voltage and corresponds to 0 dBV)

$$L_{A\ \text{Max}\ k} = 20 * \text{log}_{10}\left(\frac{A_{\text{RMS}}}{1}\right); \qquad k = 1 \ldots N \quad [\text{dB}]$$

- Decibel u-RMS: Plots the logarithmic Amplitude spectrum referred to $\sqrt{0.6}$ [Signal Unit] ($\sqrt{0.6} = 0.775$V (RMS) is a common reference level for voltage and corresponds to 0 dBu. 0.775V is the voltage that converts 1 mW electrical power on a 600 Ω resistance)

$$L_{A\ \text{Max}\ k} = 20 * \text{log}_{10}\left(\frac{A_{\text{RMS}}}{\sqrt{0.6}}\right); \qquad k = 1 \ldots N \quad [\text{dB}]$$

- Sound Pressure Level: Plots the logarithmic Amplitude spectrum referred to 20µ [Signal Unit] (20 µPa is the common reference level for sound pressure in air and corresponds to 0 dB)

$$L_{A\ \text{Max}\ k} = 20 * \text{log}_{10}\left(\frac{A\ _{\text{RMS}}}{20}\right); \qquad k = 1 \ldots N \quad [\text{dB}]$$

- Sound Pressure Level (Water): Plots the logarithmic Amplitude spectrum referred to 1µ [Signal Unit] (1 µPa is the common reference level for sound pressure in water and corresponds to 0 dB)

$$L_{A\ \text{Max}\ k} = 20 * \text{log}_{10}\left(\frac{A\ _{\text{RMS}}}{1}\right); \qquad k = 1 \ldots N \quad [\text{dB}]$$

- PSD: The Power Spectral Density (PSD) is based on the magnitude squared spectrum (M_{sq}) which differs from the amplitude squared spectrum (A_{sq}) insofar that the magnitude squared spectrum is only a one-sided spectrum.

$$M_{\text{sq}\ k} = \text{Re}\left\{Y_k\right\}^2 + \text{Im}\left\{Y_k\right\}^2; \qquad k = 1 \ldots N \quad [(\text{Unit})^2]$$

$$\text{PSD}_k = \frac{1}{N^2} * \frac{1}{\text{df}} * M_{\text{sq}\ k}; \qquad \text{with df} = \frac{\text{Samplerate}}{N} \quad \left[(\text{Unit})^2/\text{Hz}\right]$$

- PSD-TISA: plots the Time Integrated Squared Amplitude (TISA) PSD

$$PSD - TISA_k = \frac{1}{N} * \text{dt} * M_{\text{sq}\ k}; \qquad k = 1 \ldots N, \quad \text{dt} = \frac{1}{\text{Samplerate}} \quad \left[(\text{Unit})^2\, s\right]$$

- PSD-MSA: plots the Mean Squared Amplitude (MSA) PSD

$$PSD - MSA_k = \frac{1}{N^2} * M_{\text{sq}\ k}; \qquad k = 1 \ldots N \quad \left[(\text{Unit})^2\right]$$

- PSD-SSA: plots the Sum Squared Amplitude (SSA) PSD

$$PSD - SSA_k = \frac{1}{N} * M_{\text{sq}\ k}; \qquad k = 1 \ldots N \quad \left[(\text{Unit})^2\right]$$

Note: PSD, PSD-TISA, PSD-MSA and PSD-SSA are different scalings of the same spectral content and differ in the physical unit.

- Phase: Plots the phase spectrum from -180° … +180°.

$$\varphi_{\,k} = \tan^{-1}\frac{Im\{Y_k\}}{Re\{Y_k\}}; \qquad k = 1\ldots N \quad [^\circ]$$

- Phase unwrapped: Plots the unwrapped phase spectrum to avoid discontinuities from -900° … +900°.

$$\varphi_{\,k,unwrapped} = \tan^{-1}\frac{Im\{Y_k\}}{Re\{Y_k\}}; \qquad k = 1\ldots N \quad [^\circ]$$

- Phase radiant: Plots the phase spectrum from - … +.

$$\varphi_k = \frac{\varphi_k}{360^\circ}2\pi; \qquad k = 1\ldots N \quad [rad]$$

- Phase unwrapped (radiant): Plots the unwrapped phase spectrum to avoid discontinuities from – … +..

$$\varphi_{\,k,\,unwrapped} = \frac{\varphi_{\,k,unwrapped}}{360^\circ}2\pi; \qquad k = 1\ldots N \quad [rad]$$

- Weighting: allows you to apply frequency-dependent weighting to the amplitudes. The default setting is Z (none). There are also sound level weightings according to A, B, C, and D.

Section *Periodogram*

The usage of a window function damps the signal information at the window edges and emphasizes the signal information in the middle of the window function. If the signal is stationary, the variance of its spectrum rises. This problem can be avoided with a periodogram. If the option *Periodogram* is selected, the spectrum is calculated for overlapping signal parts and averaged afterwards. This procedure reduces the variance, but the spectral resolution is degraded as well.

- In the *Average* selection, the user can select the number of spectra that shall be used for the mean value calculation. 2, 3, 4, 5, 8 or 10 spectra can be used for the mean value calculation.
- In the *Overlap* selection, the user can select how much the single spectra used for the mean value calculation shall overlap in the time domain. The user can select an overlapping factor of 0 %, 50 %, 75 % 80 % or 90 %.
- The Periodogram calculation is exemplified in *Calculation of the Periodogram – Averaging of FFT windows*.

Additional instrument properties

- Frequency Axis: Change the scaling of the X-axis
- Value Axis: Change the scaling of the Y-axis. For quick Y-axis scaling features, refer to *Quick selection Y-axis scaling*.
- Style:
  - Selection of a transparent or untransparent background.
  - Line Width selection from 1…10
  - Show short channel name. This option does not display the node or group channel name in case the channel name has one. "AI 1/1@DEWE3-RM16" will be displayed as "AI 1/1" with the activated option.
- Layer: Moves the Instrument in front of or behind another object

Note: The properties of the FFT can be changed and updated in the *PLAY* mode as well as in the *LIVE* and *REC* mode.

8.12.5 Markers

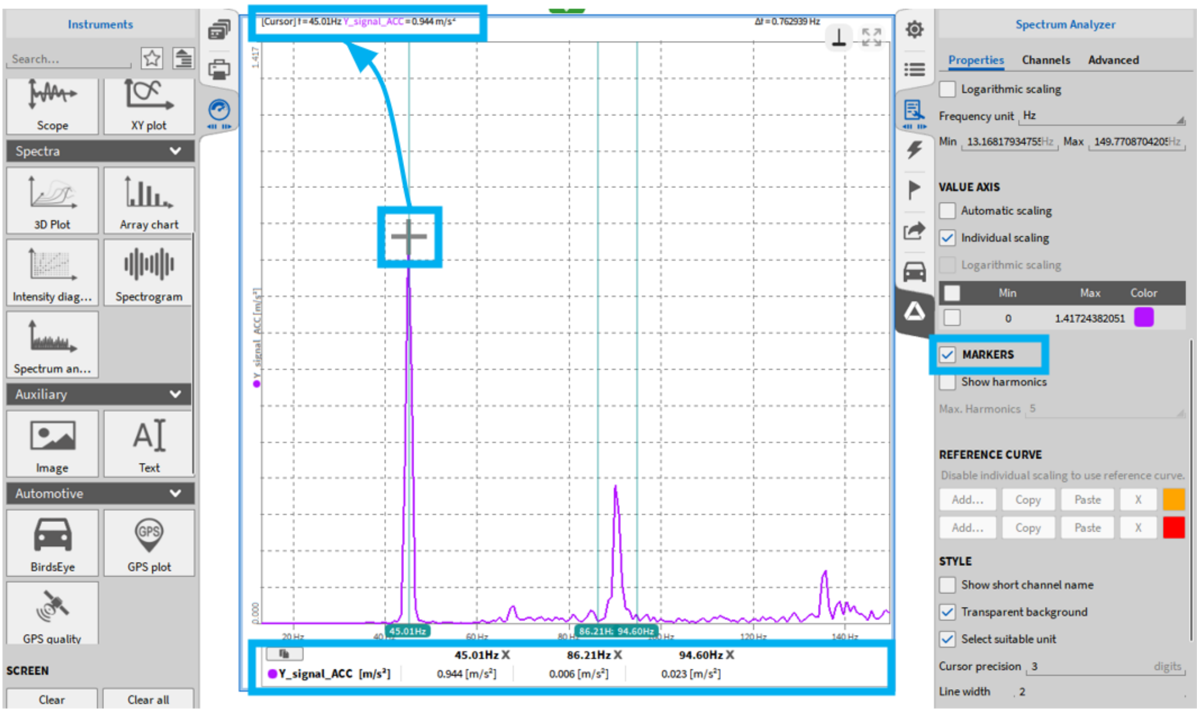


Fig. 8.51: FFT Marker - Overview

To analyze the behavior of a certain frequency line, the user can display the actual value in a table below the FFT plot. Therefore, the user must activate MARKERS with the respective checkbox in the instrument settings and select the desired frequency line with a mouse click afterwards. Then, the selected point will show up in the table. The user can change the frequency position by moving the respective cursor across the frequency axis or with a double click on the frequency in the table. Up to five frequency lines can be displayed in the table simultaneously. While moving the mouse in the frequency plot, the actual frequency and the actual signal value of the signal next to the cursor are displayed in the upper left corner. When markers are set and the respective checkbox in the instrument settings will be deactivated, the set markers stays untouched, but it is not possible to add new markers until the MARKERS checkbox will be activated again.

8.12.6 Usage Of Harmonics Cursors

Harmonics Cursors can be displayed by checking *Show Harmonics* (see ① in Fig. 8.52). The number of harmonics can be set from 1 to 10 (see ② in Fig. 8.52). Harmonics are marked with cursors (see ③ in Fig. 8.52) and the harmonics amplitude is displayed at the instrument's bottom (see ④ in Fig. 8.52).

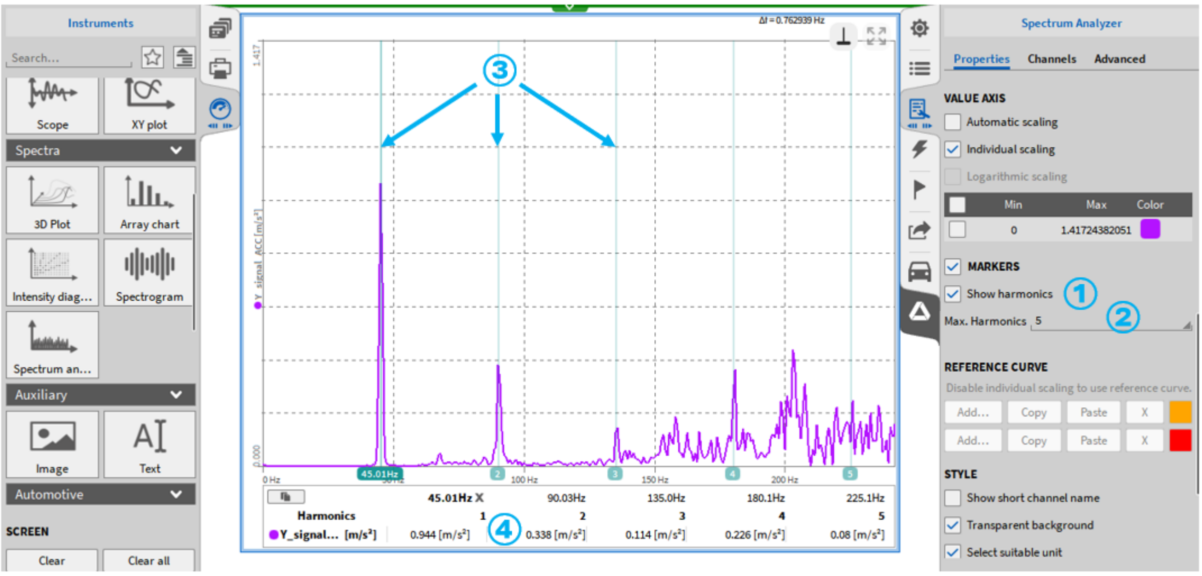


Fig. 8.52: Usage of Harmonics Cursors

The cursor position can be changed by entering a new frequency for the first harmonic (see ⑤ in Fig. 8.53). It is also possible to move the first harmonic cursor with the left mouse button (see ⑥ in Fig. 8.53). The position of the higher harmonics is automatically adjusted.

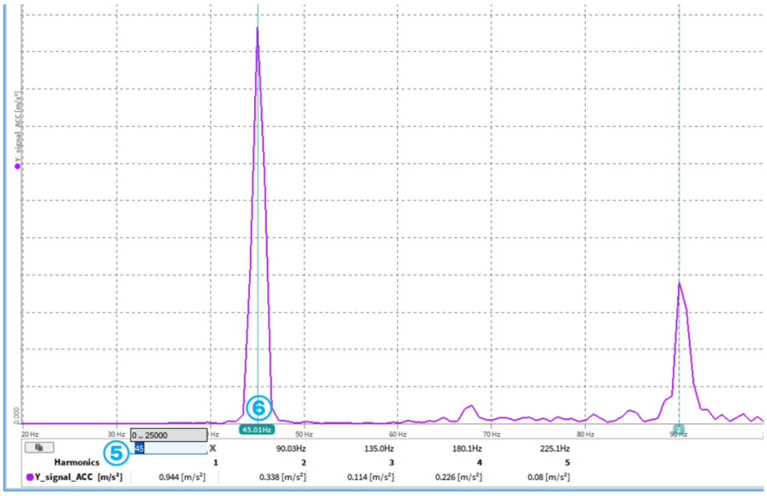


Fig. 8.53: Changing the 1st Harmonics cursor position

8.12.7 Reference Curves for the Spectrum Analyzer

The Spectrum Analyzer provides the possibility to create reference curves for threshold monitoring in the frequency domain.

An orange and a red colored reference curves can be created which will colorize the instruments' background orange or re if the signal exceeds the reference curve.

The red reference curve has a higher priority than the orange one. This means that the instruments' background will be colored red if the threshold of both reference curves will be exceeded. The colored background will be reset automatically when the threshold is decreased again.

To create a Reference curve, press the *Add..* button in the Reference Curve section of the Spectrum Analyzers' Instrument Properties (see Fig. 8.54). If the Linear interpolation checkbox is enabled, the set X and Y values are interpolated.

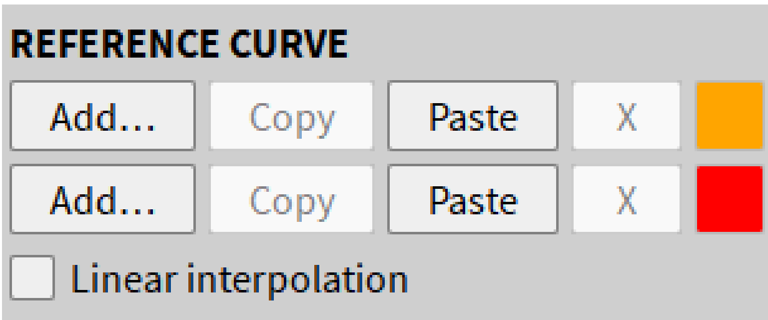


Fig. 8.54: Instrument properties for Reference curves

A popup menu will open and the reference curve can be set up in table form (see Fig. 8.55). The *+* button can be used to add a value. If the Linear interpolation checkbox is enabled, the set X and Y values are interpolated.

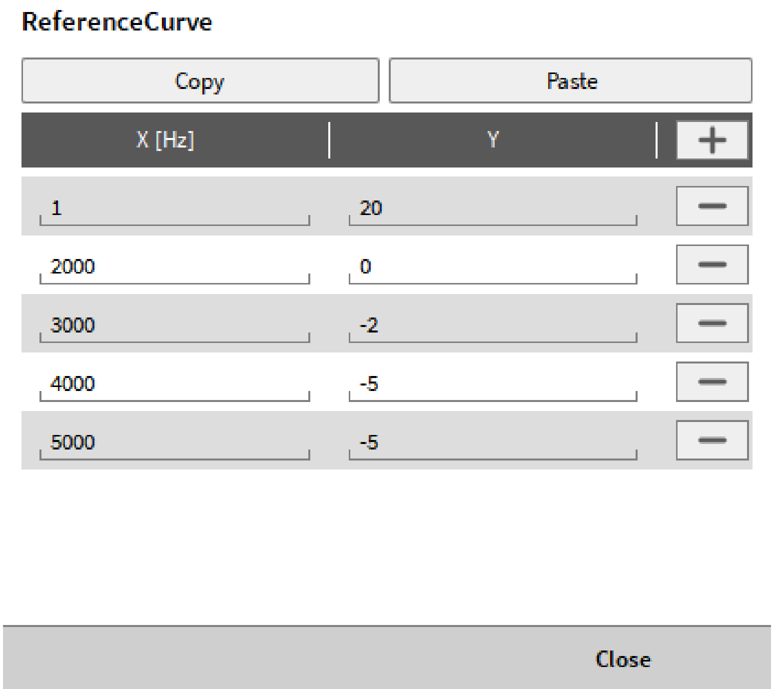


Fig. 8.55: Table for reference curves definition

The following Fig. 8.56 and Fig. 8.57 demonstrate the steps to create an orange and a red reference curve:

1. Click on the *Edit...* button
2. Press *+* to add one or more lines to the table
3. Enter the frequency and the corresponding reference value to the table
4. Press *Close* when finished and the curve will instantly be displayed

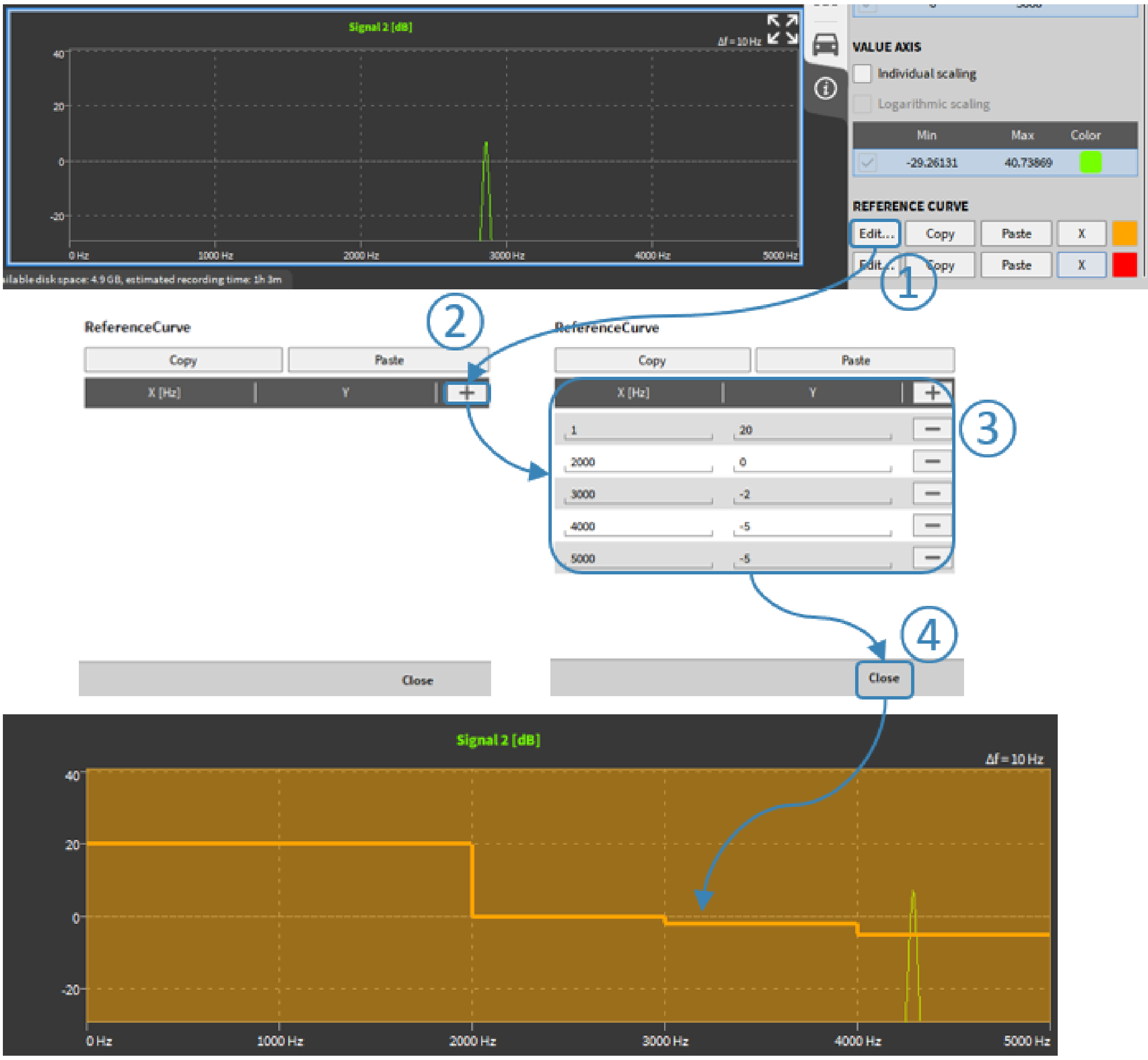


Fig. 8.56: How to create an orange reference curve

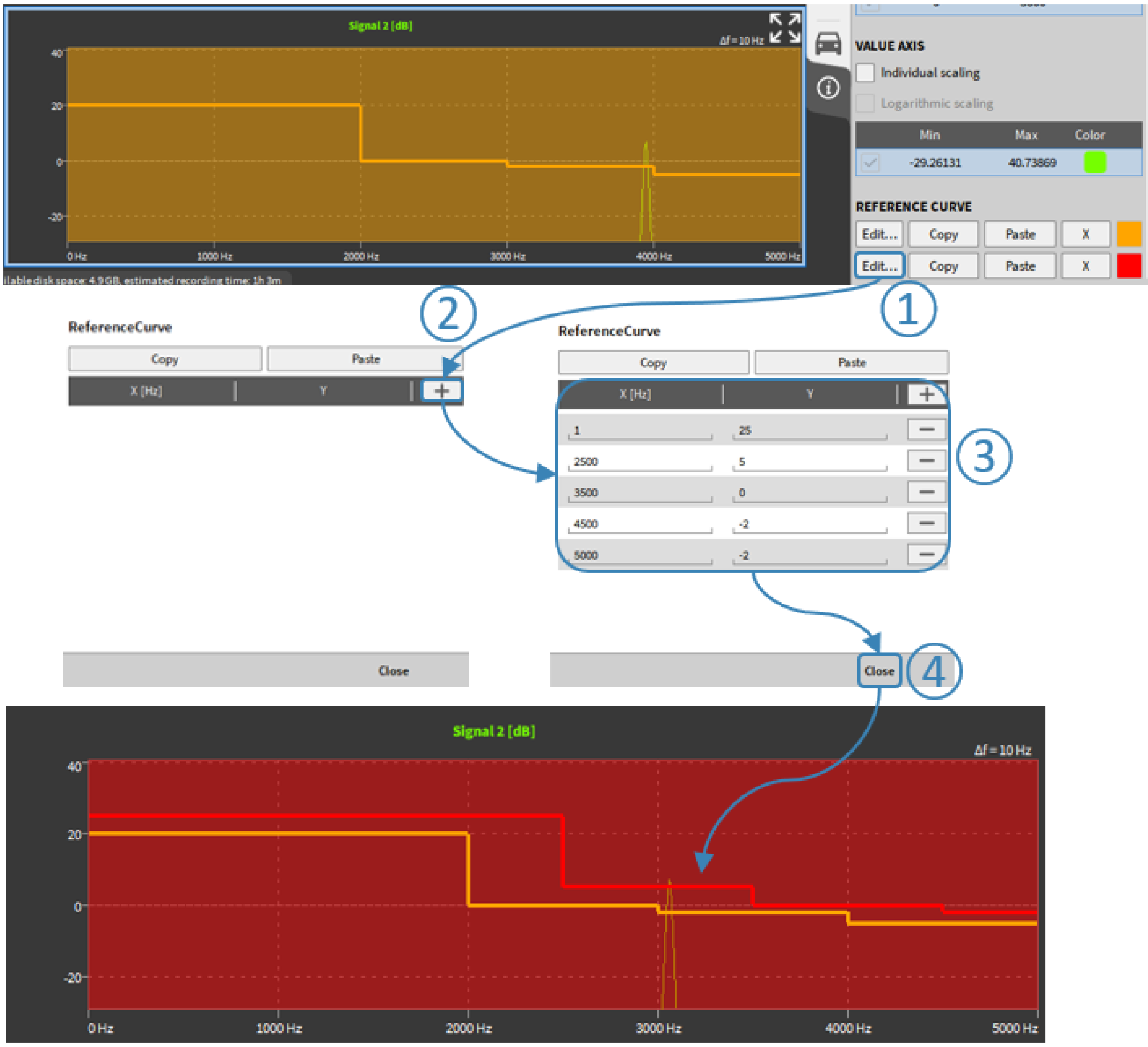


Fig. 8.57: How to create a red reference curve

The *Copy* and *Paste* buttons can be used to copy and paste the table from the orange to the red curve and vice versa (see Fig. 8.58) or to export and import a value table into / to clipboard for interacting with Excel or other 3rd party software (see Fig. 8.59).

The *X* button (see Fig. 8.54) can be used to delete a reference curve again.

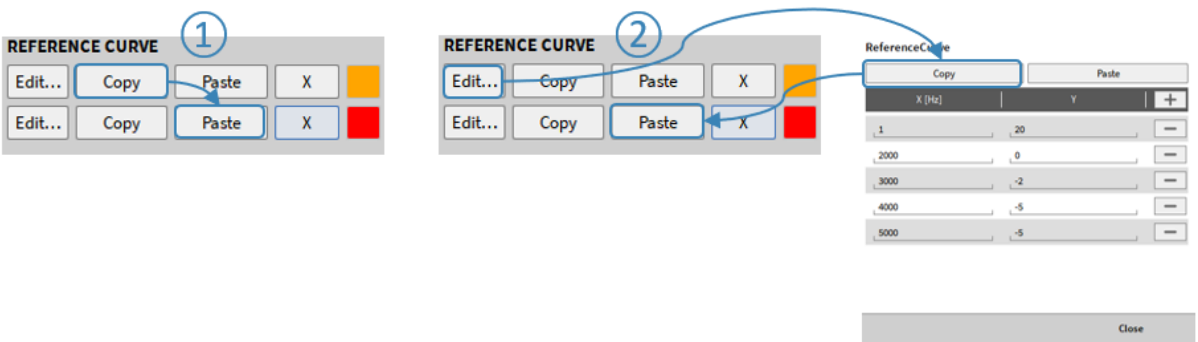


Fig. 8.58: Copy and paste settings from one reference curve to another

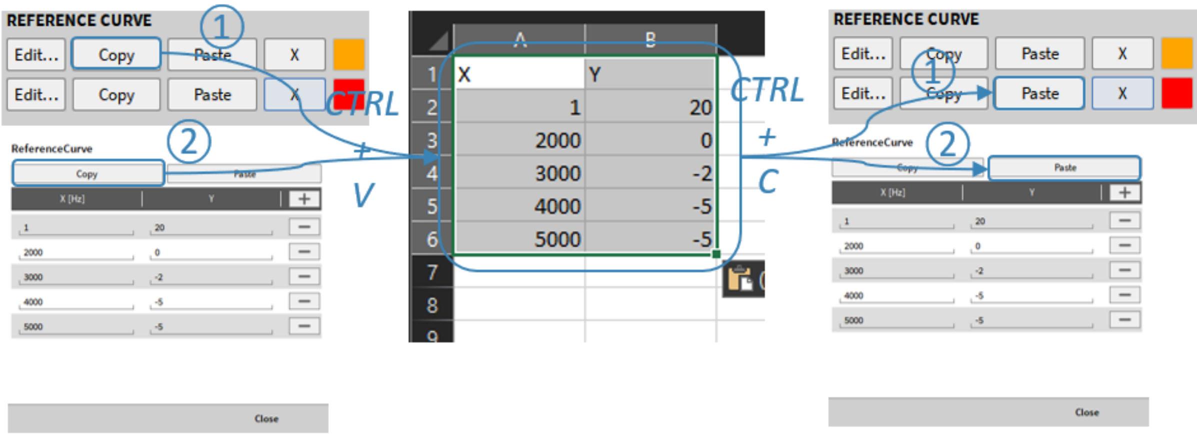


Fig. 8.59: Copy and paste values from/into Excel

As soon as the table has been set up, the reference curve will be displayed in the Spectrum Analyzer (see Fig. 8.60, Fig. 8.61 and Fig. 8.62).

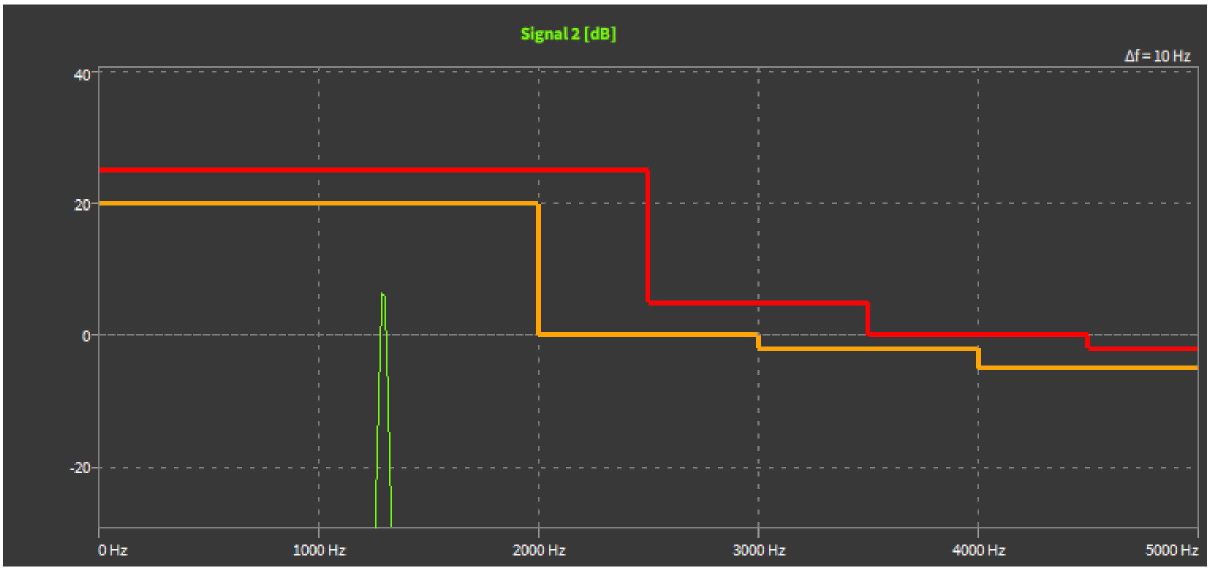


Fig. 8.60: Reference curves without limit exceeded

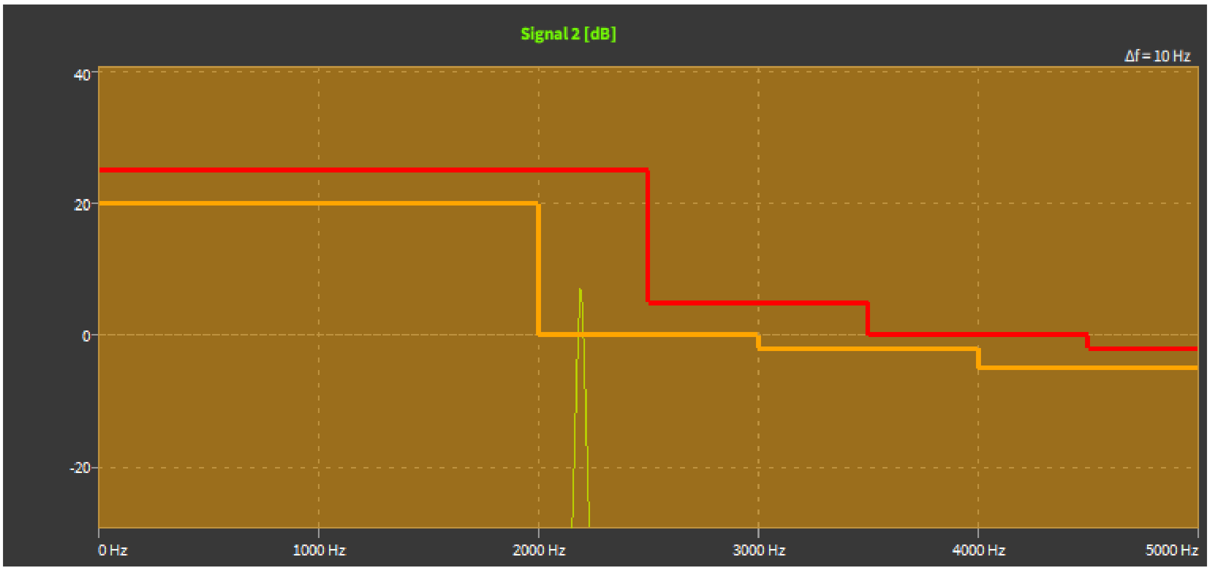


Fig. 8.61: Reference curves with orange limit exceeded

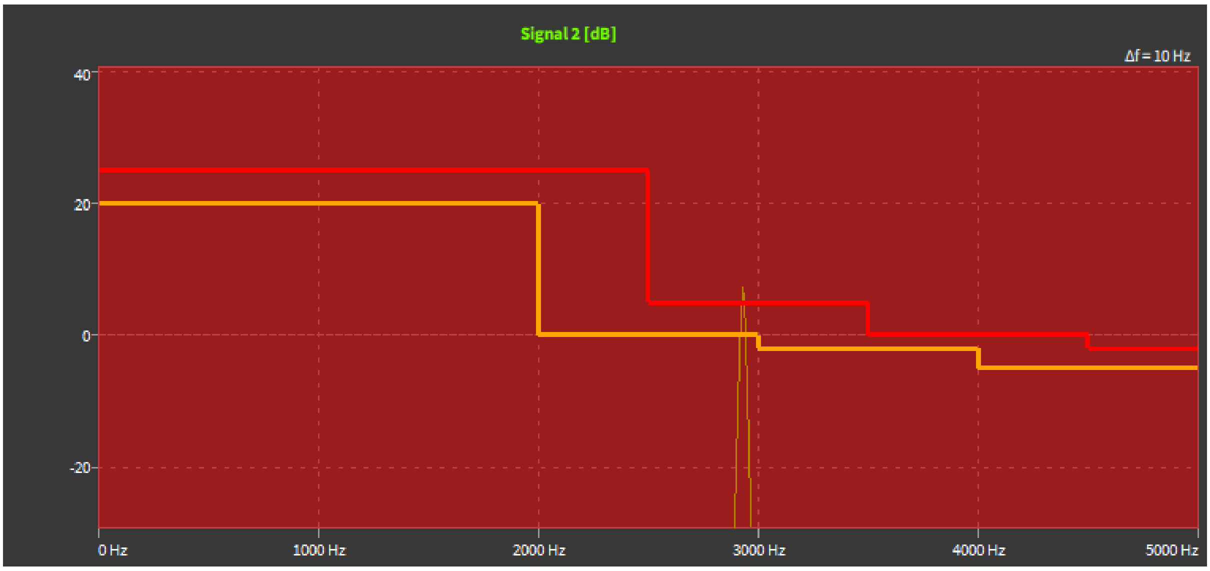


Fig. 8.62: Reference curves with orange and red limit exceeded

8.12.8 Cross hairs

There are 2 options for crosshairs: *Use crosshair cursor* and *Use peak cross hair*. Additionally the Line width of the cross hair can be defined.

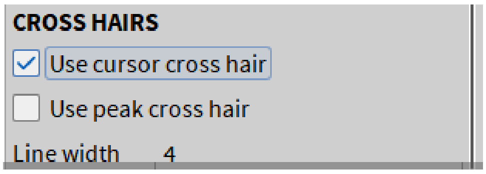


Fig. 8.63: Cross hairs option of the spectrum analyzer

With the “Follow Peak” function at the Crosshairs option, the peak value in the visible area of the FFT instrument is visually marked with the help of a crosshair (see Fig. 8.64). The cross hairs jumps automatically to the highest peak, which makes it easy to recognize.

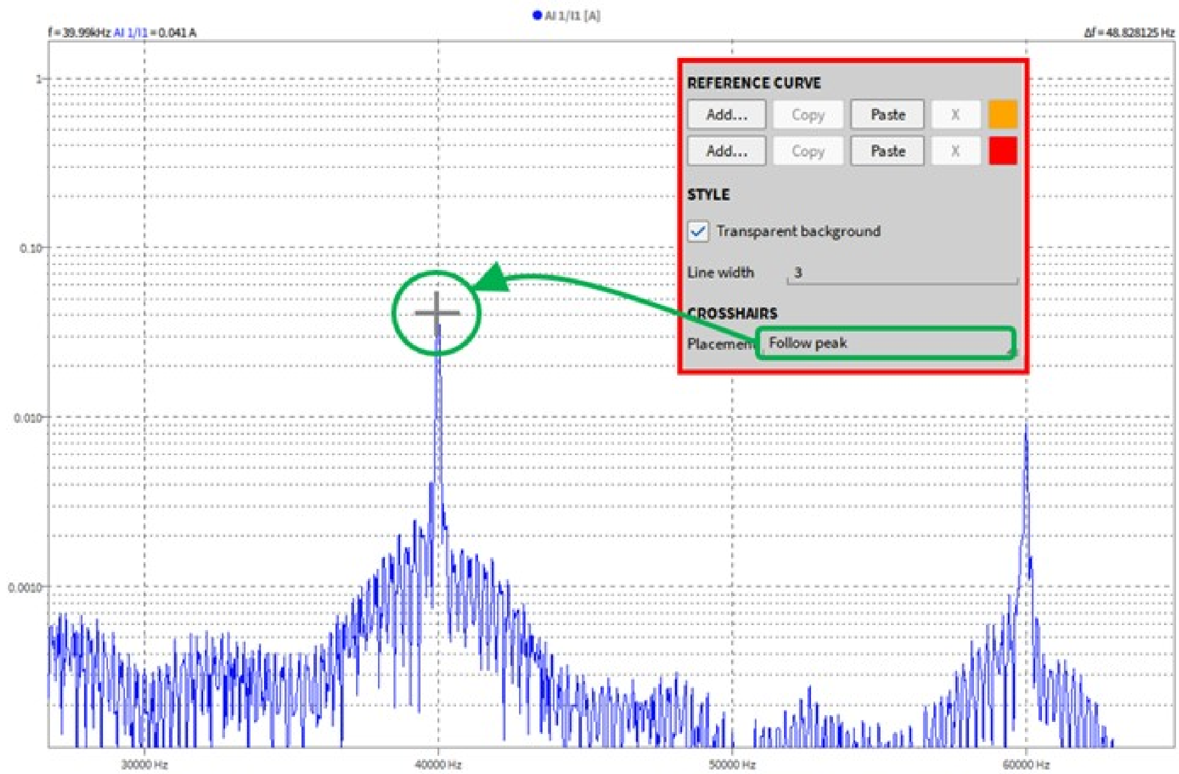


Fig. 8.64: Follow Peak

8.12.9 FFT for recorder region

It is also possible to calculate the FFT for the assigned time domain channel based on a selection from A/B cursor in a recorder. For this to work, the recorder needs to be on the same page and has its settings to “Link mode: Instruments on page” (①). The channel of the recorder must be also assigned to the spectrum analyzer and the FFT option “Link to Recorder Cursor” must be enabled (②).

Fig. 8.65: Spectrum analyzer with data based on recorder region

This function is available in LIVE (freeze) and PLAY mode.

8.12.10 Additional information for the spectrum analyzer properties

Further explanations on line resolution, normalization, and averaging are provided below.

Additional information: improve line resolution (Enable zero-padding)

If *Improve Line Resolution* is selected, zero-padding is enabled. The following paragraph explains the idea of zero-padding and its properties.

Theory of zero-padding

If zero-padding is not applied, the line resolution and thus the accuracy of a FFT depends on the length of the transformed signal and on the sample rate:

$$Line\ Resolution = \frac{\text{Samplerate}}{\text{Window size}} [Hz]$$

The data size is equal to the number of FFT bins here. Thus, a higher line resolution can be achieved by reducing the sample rate or increasing the data size. Normally, a sample rate reduction cannot be accepted due to bandwidth reasons. Increasing the data size may cause problems in Realtime applications, because the delay until an FFT is displayed increases with increasing data size. Moreover, if short signals are transformed, a data size increment is simply not possible.

Zero-padding adds zeros at the end of the signal part to be transformed and thus increases the data size artificially. Please note that the *Data size* is not any more equal to the number of FFT bins. The following example will clarify that: A 64-sample signal in time domain shall be matched to an FFT with 256 FFT bins. Therefore, 192 zeros must be added at the end of the 64-sample signal in time domain. Thus, the Line resolution can be determined according to the following formula:

$$Line\ Resolution = \frac{\text{Samplerate}}{Window\ size + Number\ of\ zeros} = \frac{\text{Samplerate}}{\text{Number of frequency lines}}\ [Hz]$$

In OXYGEN, the number of attached zeros can be manipulated indirectly by varying the *Data size* or the *Line resolution* in the Instrument Properties of the Spectrum analyzer (see *FFT properties for Time Domain Channels*).

In OXYGEN, the Line resolution can be selected from $\frac{\text{Samplerate}}{2^{20}}$ to $\frac{\text{Samplerate}}{\text{Window size}}$ if zero-padding is selected. If a lower line density is desired, zero-padding is not required and can be de-selected.

In the signal theory, the two most common application areas of zero-padding are the already explained increased sample density in the frequency domain and the signal enlargement to a length of 2^n samples, because time signals with a length of 2^n samples permit a faster FFT-computation.

Even though zero-padding increases the sample density in the frequency domain, the FFT is not more accurate if zero-padding is used. Zero-padding is only a kind of an interpolation and does not increase the resolution. This characteristic is shown in *Zero-padding – A practical example*. To increase the resolution, a longer signal in time domain is required.

Note: Zero-padding is applied after multiplying the signal with the window function.

Zero-padding – A practical example

In this section, zero-padding is explained with an easy practical example. For this purpose, the following signal is used:

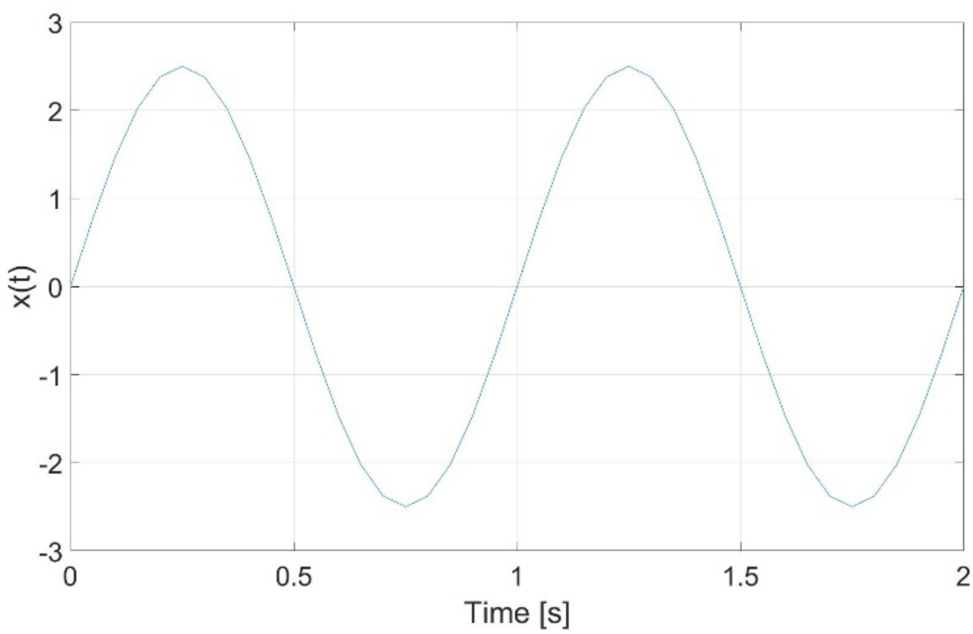


Fig. 8.66: Signal 1 in time domain, 2s (41 samples)

$$x(t) = 2.5 * sin(2 * \pi * 1 * t)$$

The signal has a length of 2 seconds and is sampled with 20 Hz. Thus, the signal consists of 41 samples. Transforming the signal into the frequency domain leads to the following spectrum:

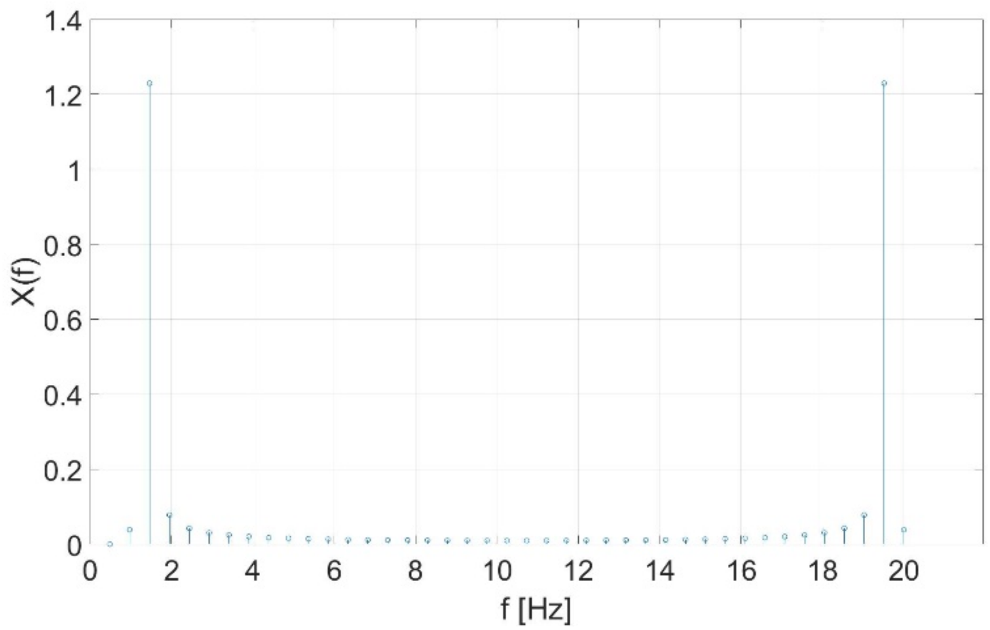


Fig. 8.67: Signal 1 in frequency domain, no zero-padding

The spectrum consists of 41 bins and the peaks @1 Hz and 19 Hz are clearly visible.

Now, the signal length is enhanced from 41 samples to 64 samples by adding 23 samples at the end of the signal:

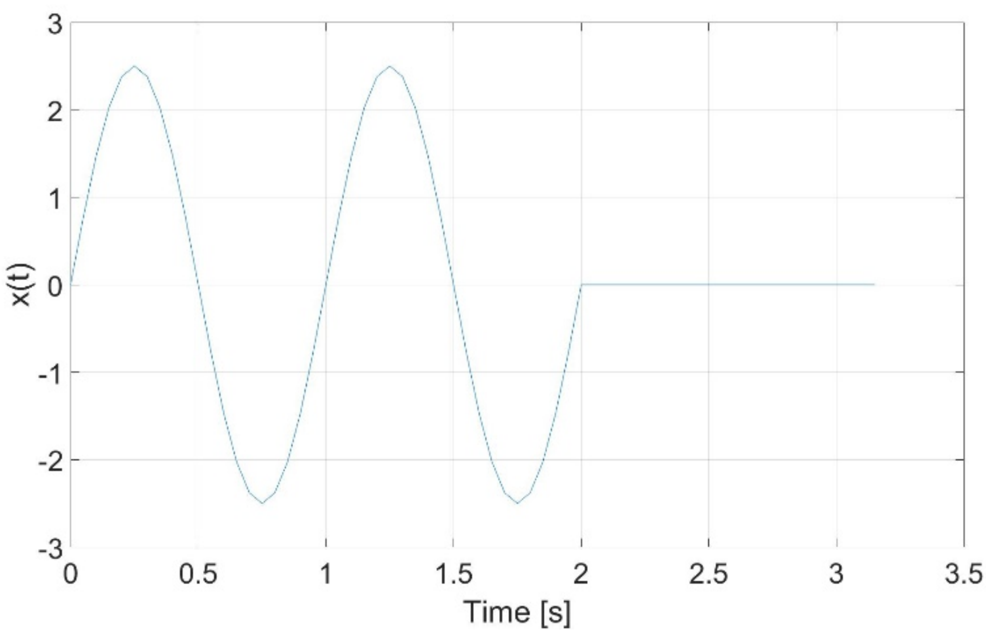


Fig. 8.68: Signal 1 in time domain, zero-padding to 64 samples

Transforming the signal to the frequency domain leads to the following spectrum:

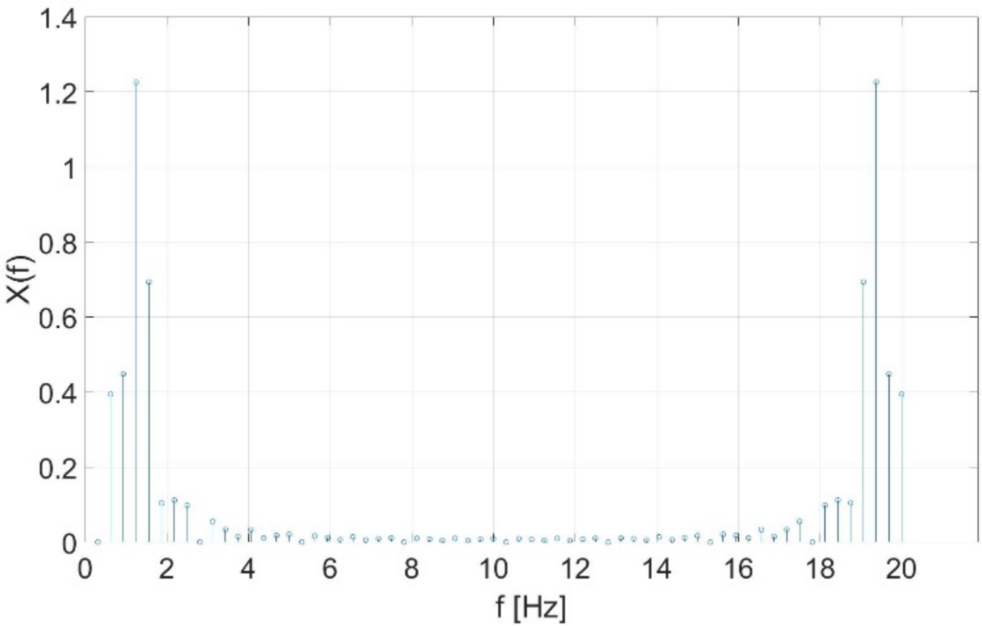


Fig. 8.69: Signal 1 in frequency domain, zero-padding to 64 samples

Now the spectrum consists of 64 samples and not 41 samples and the additional frequency bins are kind of an interpolation but do not lead to a sharper spectrum.

The same trend is visible if the original signal is enhanced from 41 samples to 128 samples by adding 87 zeros at the end of the signal:

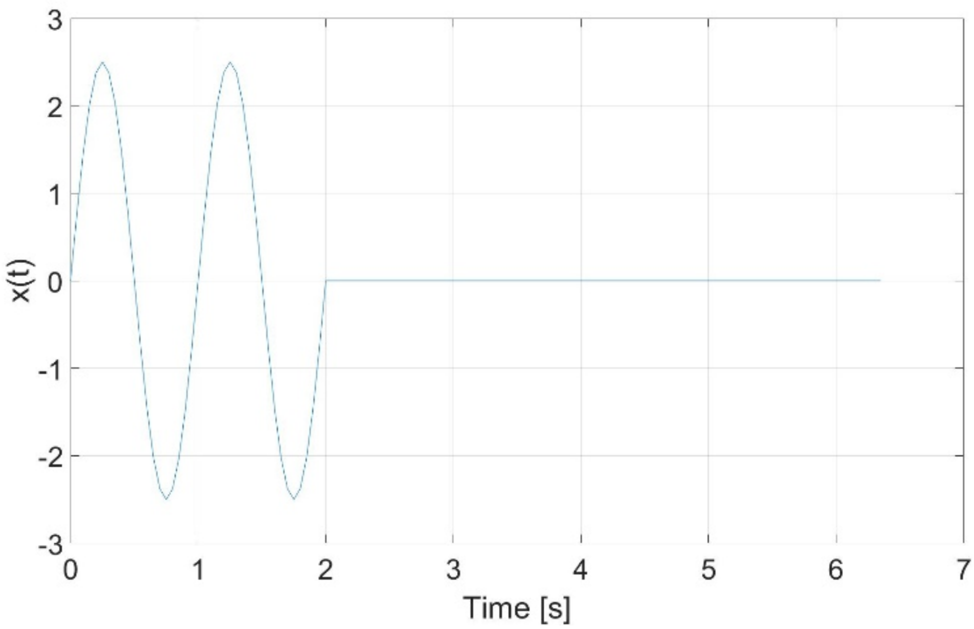


Fig. 8.70: Signal 1 in time domain, zero-padding to 128 samples

This signal leads to the following spectrum with 128 frequency bins:

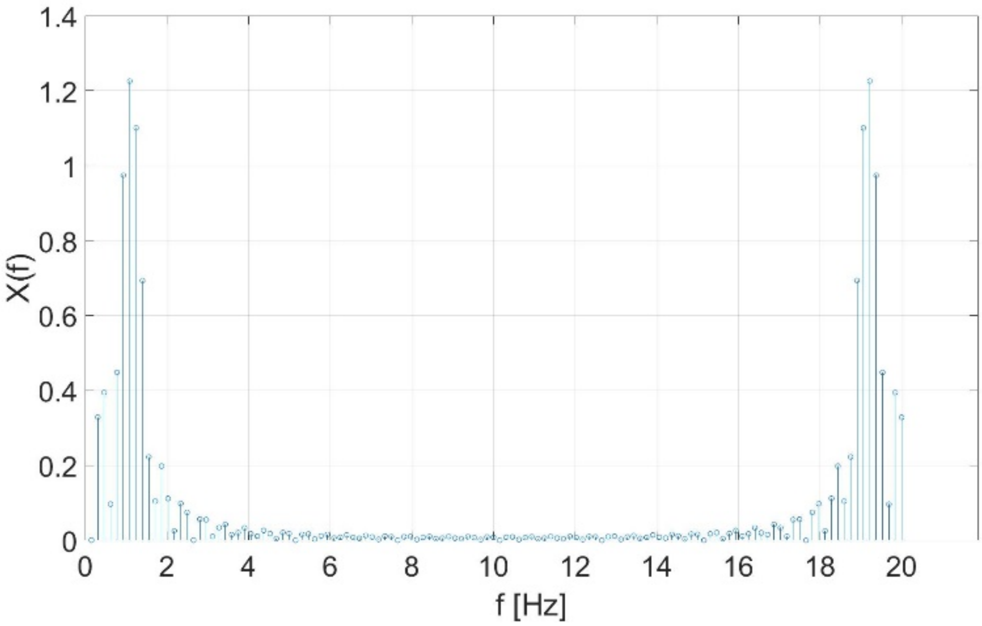


Fig. 8.71: Signal 1 in frequency domain, zero-padding to 128 samples

Again, the additional bins are only kind of an interpolation, but do not lead to a sharper spectrum.

To enlarge the accuracy of the FFT, a longer signal in time domain is required. Therefore, the original sine signal is enlarged to 6.4 seconds (128 samples):

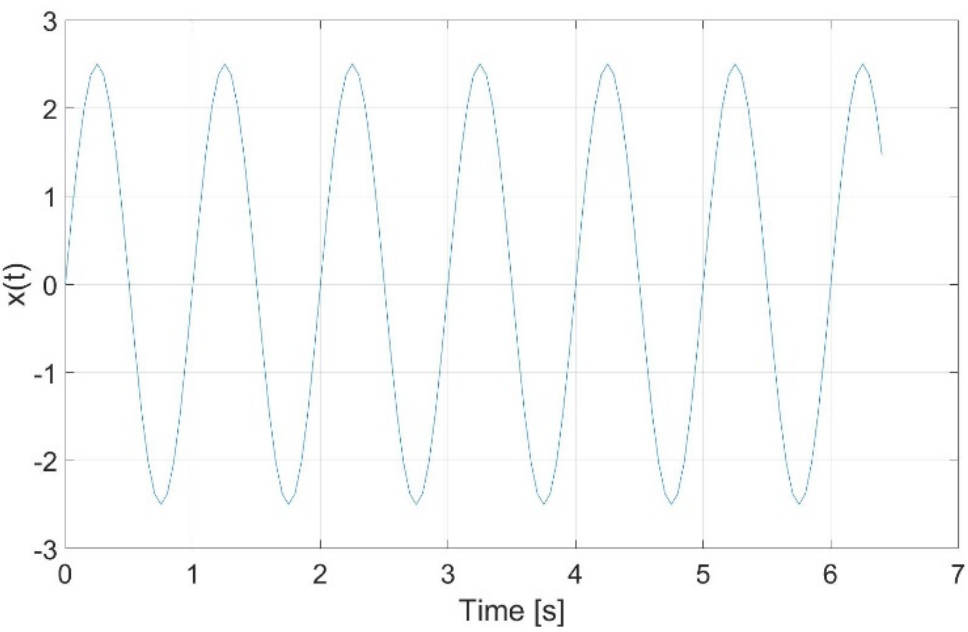


Fig. 8.72: Signal 2 in time domain, 6.4s (128 samples)

The resulting spectrum consists also of 128 bins but now, the additional bins really lead to a sharper spectrum and are no longer only an interpolation of the original 41 frequency bins:

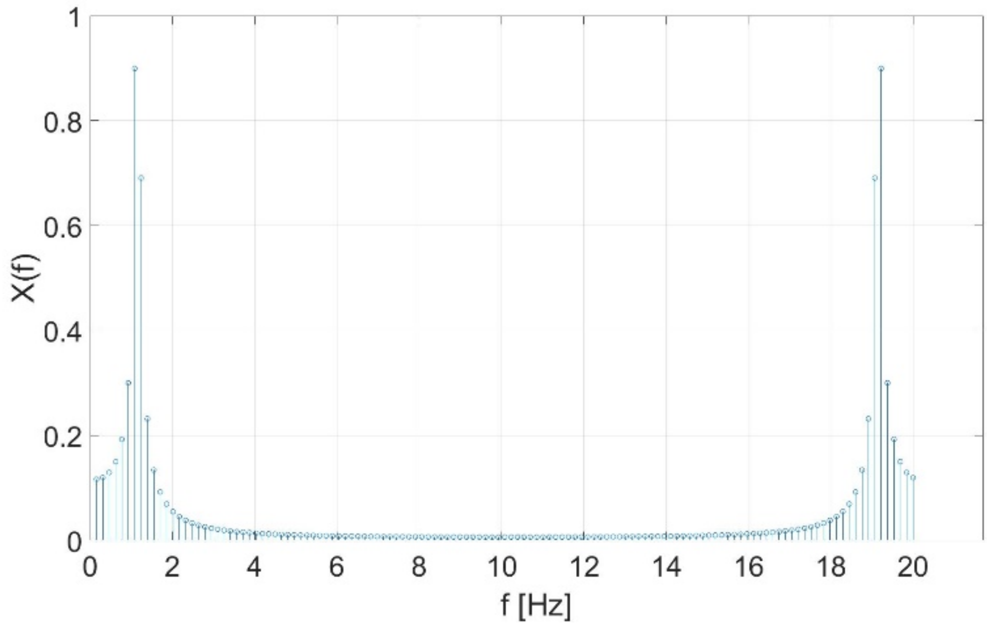


Fig. 8.73: Signal 2 in frequency domain, no zero-padding

Normalization of FFT Spectra

In this section, the necessity of the normalization during the FFT calculations is explained. Therefore a 50 Hz sine wave with 2.5 amplitude shall be transformed to the frequency domain. The sample rate is 1000 Hz and the signal length 10s. The signal looks as follows in time domain:

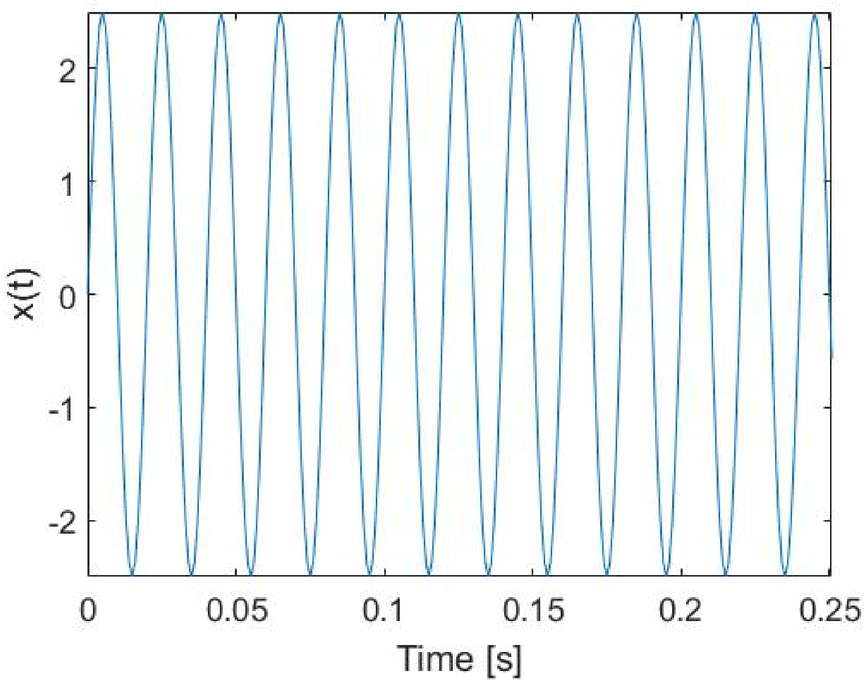


Fig. 8.74: Signal in time domain (first 250 ms)

$$x(t) = \ 2.5 * sin(2 * \pi * 50 * t)$$

After transforming the signal into the frequency domain according to the formula

$$Y_k = \sum_{n=0}^{N-1} X_k e^{\frac{-i2\pi kn}{N}}; \qquad k = 0 \ldots N-1 \qquad (N = 10001)$$

and determining the absolute value, the spectrum is the following:

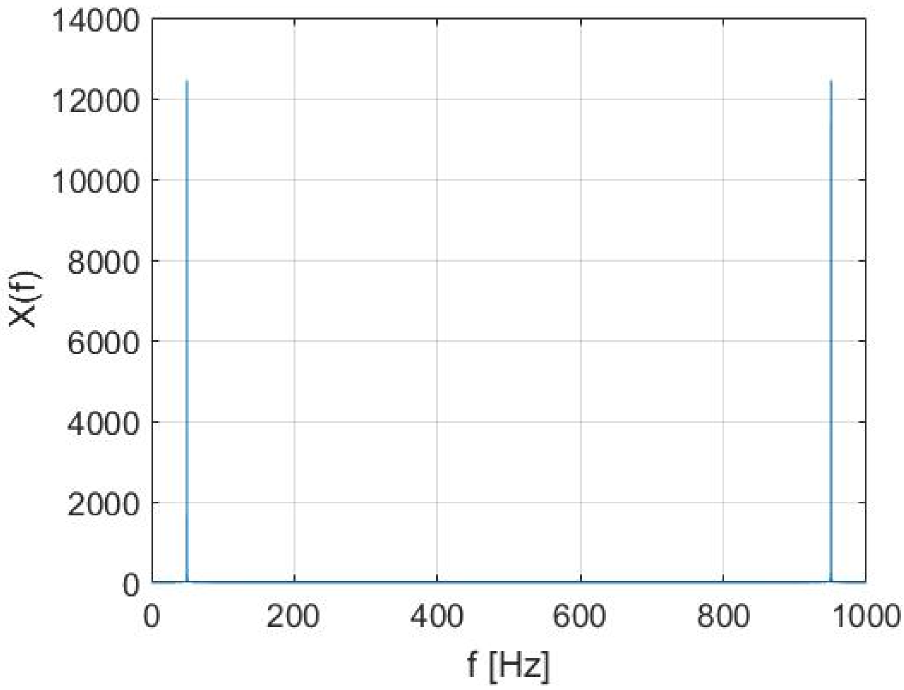


Fig. 8.75: x(t) in frequency domain

Two things are peculiar:

- As the FFT produces a two-sided spectrum, there is a bin @ 50 Hz and @ 950 Hz.
- As the signal level of the two peaks is ~12500, the unit seems to be arbitrary.
- To create a comprehensible signal unit, the Fourier Transform of the signal must be divided by the length of the FFT which is 10001 in this example.

$$Y_{\text{norm}_k} = \frac{Y_k}{N}; \qquad k = 0 \ldots N-1 \quad (N = 10001)$$

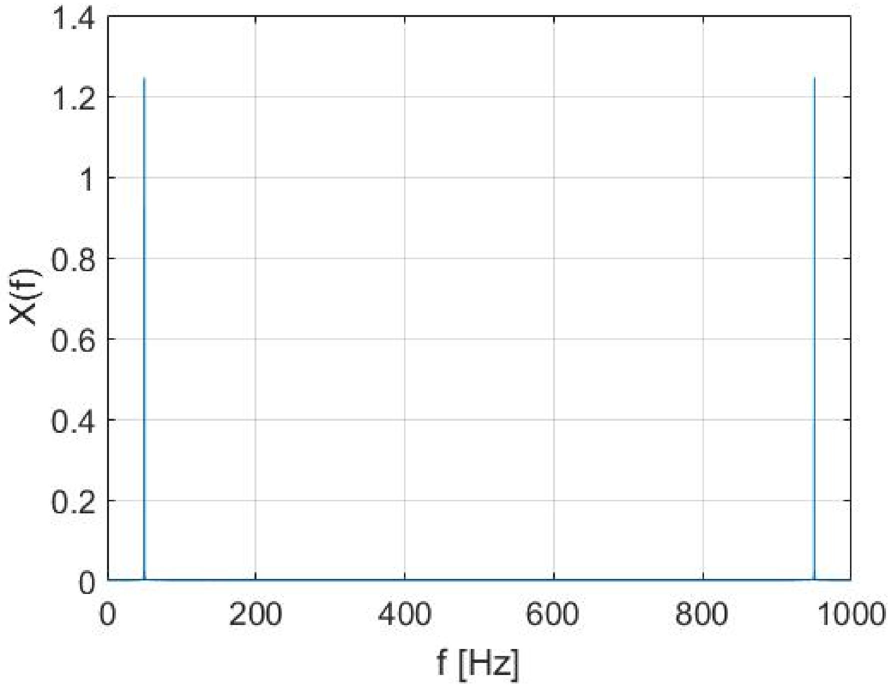


Fig. 8.76: x(t) in frequency domain divided by the FFT-length

Now, the amplitude of both peaks is ~1.25. As we still have two peaks whose sum is ~2.5, the signal unit issue is solved by dividing the spectrum by the length of the FFT.

In a next step, we truncate the spectrum at the Nyquist frequency ($\left(\frac{f_s}{2}\right)$) which is 500 Hz in our case and multiply the remaining spectrum from 0 to 500 Hz with the factor 2 to ensure that the power of the signal in the frequency domain is still the same as in the time domain. After that, the following spectrum results:

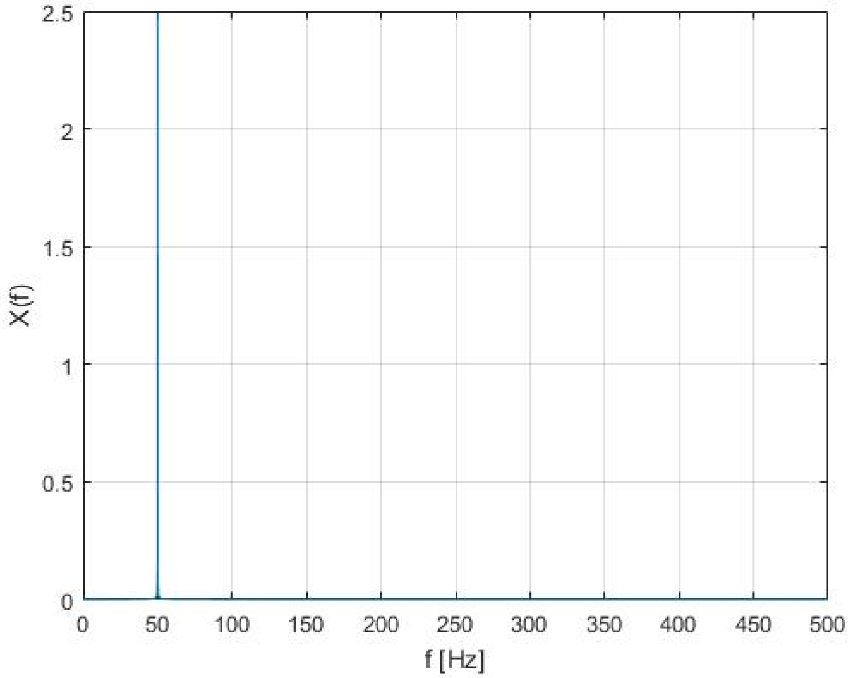


Fig. 8.77: One-sided spectrum X(f) multiplied by factor 2

In this first example, there is no normalization needed, because we didn't use a window function. In this case, there was no window function needed, because we transformed a finite and periodical signal. In practice, this is normally not the case and a continuing signal is transformed block by block. As these block lengths are finite, the Leakage effect occurs if the block length does not coincidentally match with an integer multiple of the signal period. In this case, the frequency spectrum becomes too wide. This is a natural effect resulting from the Fourier Transform property which says that a multiplication in time domain leads to a convolution in the frequency domain. The fact that the frequency spectrum becomes too wide can be optimized but not completely rejected by the usage of a window function. This leads to the fact that the signal is faded in at the beginning of the window and faded out at the end of the window. Thus, an artificial periodical signal results and an error in the signal amplitude results. This amplitude error is corrected by the normalization of the signal.

Let's assume again the 50 Hz sine wave with 2.5 amplitude shown in Fig. 8.74 and multiply it with a Hanning window. The formula for the creation of a Hanning window can be found in section *Window type*. After the multiplication, the signal looks as follows:

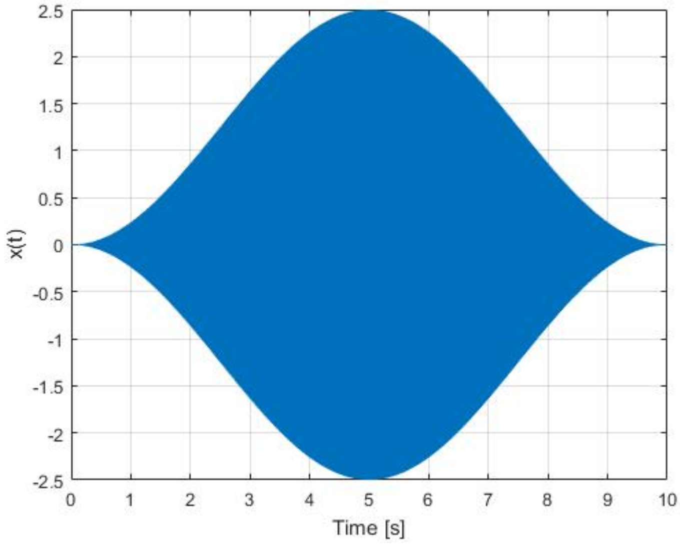


Fig. 8.78: $x(t)_{win}$ in time domain; multiplied with a Hanning window

$$x(t)_{\text{win}} = [2.5 * sin(2 * \pi * 50 * t)] * \left[0.5 * \left(1 - \cos\left(\frac{2 * \pi * n}{N - 1}\right)\right)\right]; \quad n = 0 \ldots N - 1$$

The spectrum of the signal looks as follows:

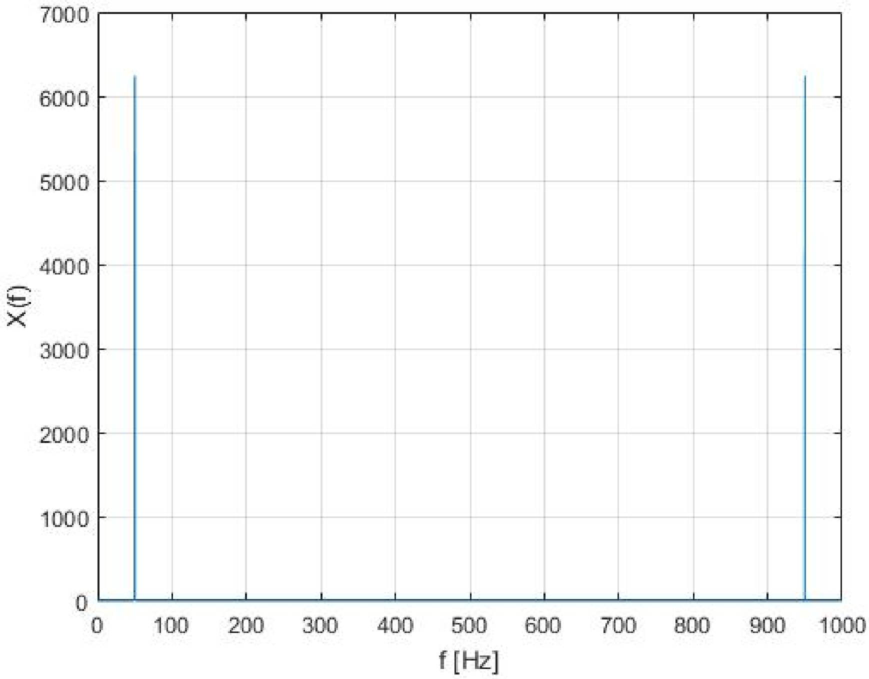


Fig. 8.79: $x(t)_{win}$ in frequency domain

Again, the signal unit looks arbitrary. Thus, we divide the spectrum by the length of the FFT (N=10001) again.

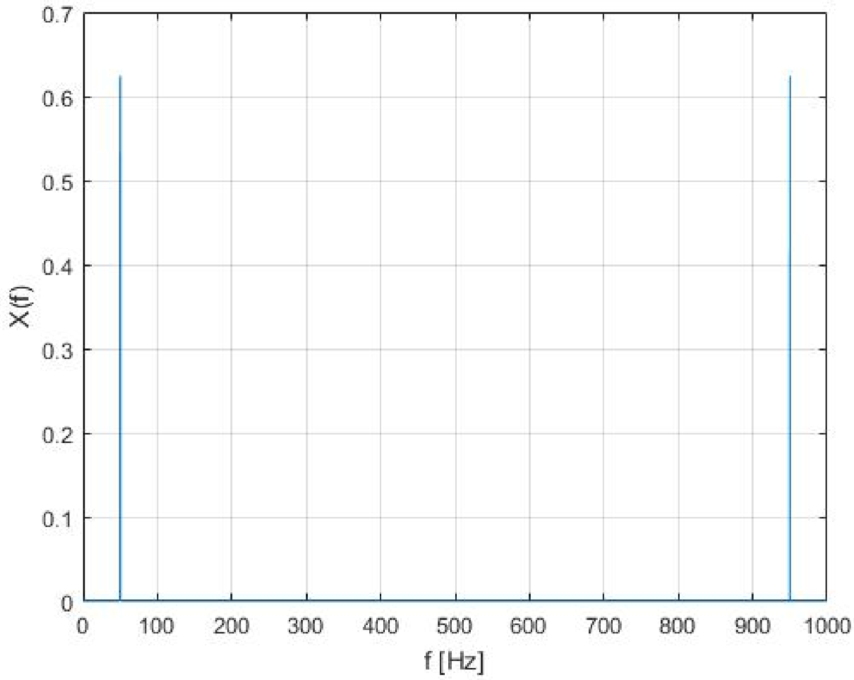


Fig. 8.80: $x(t)_{win}$ in frequency domain divided by the FFT-length

After that we truncate the signal again at the Nyquist frequency and multiply the remaining spectrum with the factor 2 to secure that the signal power in time and frequency domain is equal.

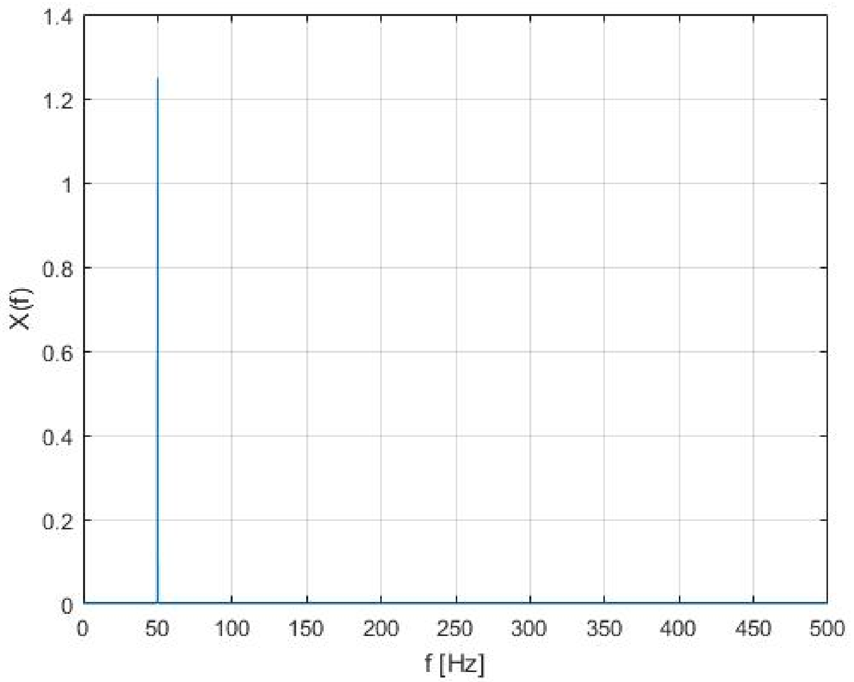


Fig. 8.81: One-sided spectrum $X(f)_{win}$ multiplied by factor 2

Now we clearly see that the peak @50 Hz is not 2.5 as before but only ~1.25. This is because of the windowing. This can be corrected with the normalization. There are two possibilities: We can either normalize the spectrum to the original signal amplitude or to the original signal power.

To refit the spectrum according to the original signal amplitude, we must select the *Amplitude True* normalization:

$$X(f)_{\text{win}_{\text{AmpCorr}}} = X(f)_{\text{win}} * [\frac{N}{\sum_{k=1}^{N} W_k}]$$

where N denotes again the window (and signal) length and W_k the value of the window function at position k.

There we can see that the peak @50 Hz is again 2.5. But in this case, the signal power in frequency domain is not the same as in time domain. If this is required, we must select the *Power True* normalization:

$$X(f)_{\text{win}_{\text{PowCorr}}} = X(f)_{\text{win}} * \sqrt{\frac{N}{\sum_{k=1}^{N} W_k^2}}$$

where N denotes again the window (and signal) length and W_k the value of the window function at position k.

Now, the power in frequency domain is the same as in time domain, but the amplitude does not match correctly anymore.

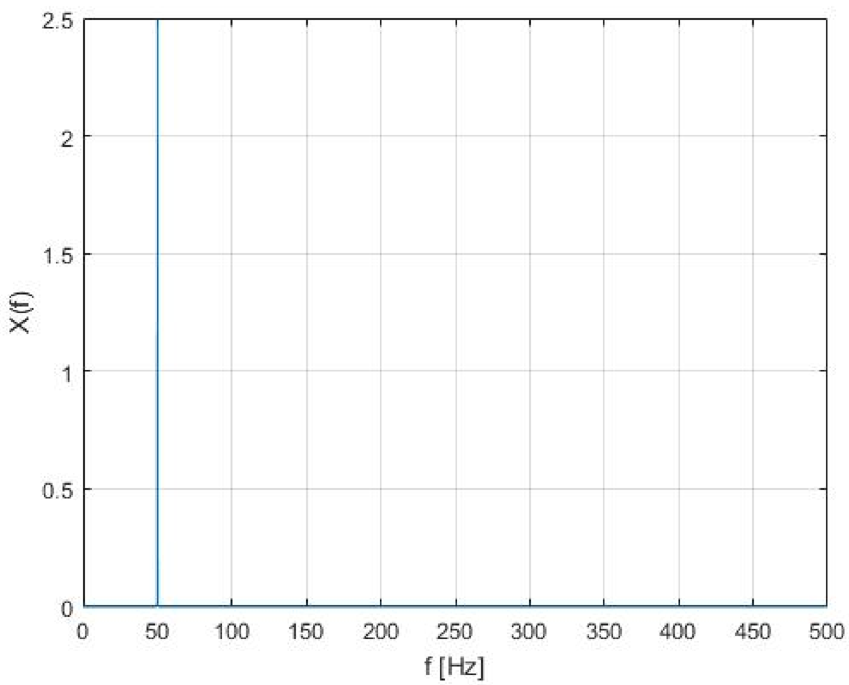


Fig. 8.82: Amplitude-True-normalized spectrum X(f)

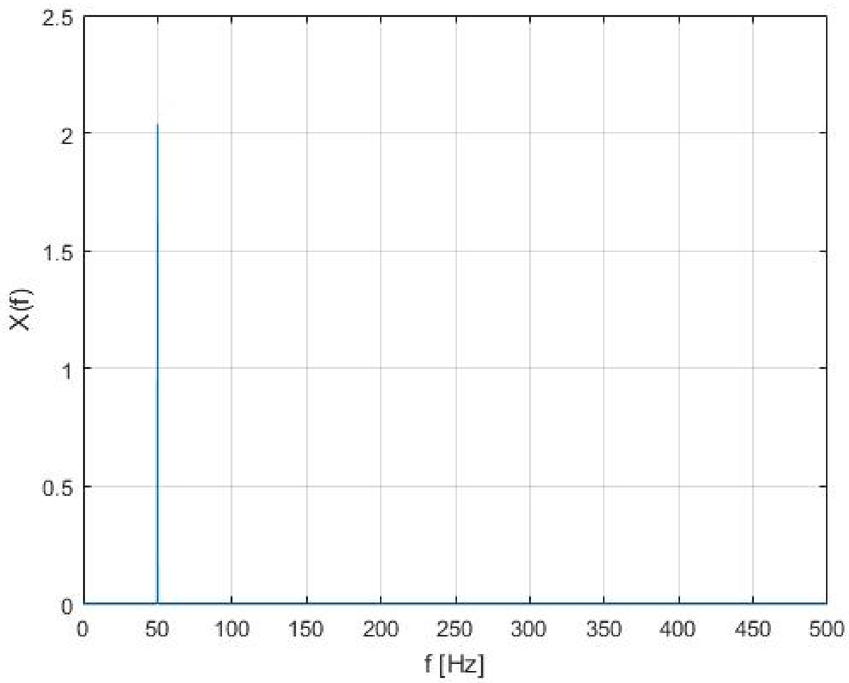


Fig. 8.83: Power-True-normalized spectrum X(f)

Calculation of the Periodogram – Averaging of FFT windows

This section will demonstrate the calculation of a periodogram in a practical example. The exemplary window size is 1000 samples. The following figures illustrate the decomposition of a time signal for the calculation of a periodogram:

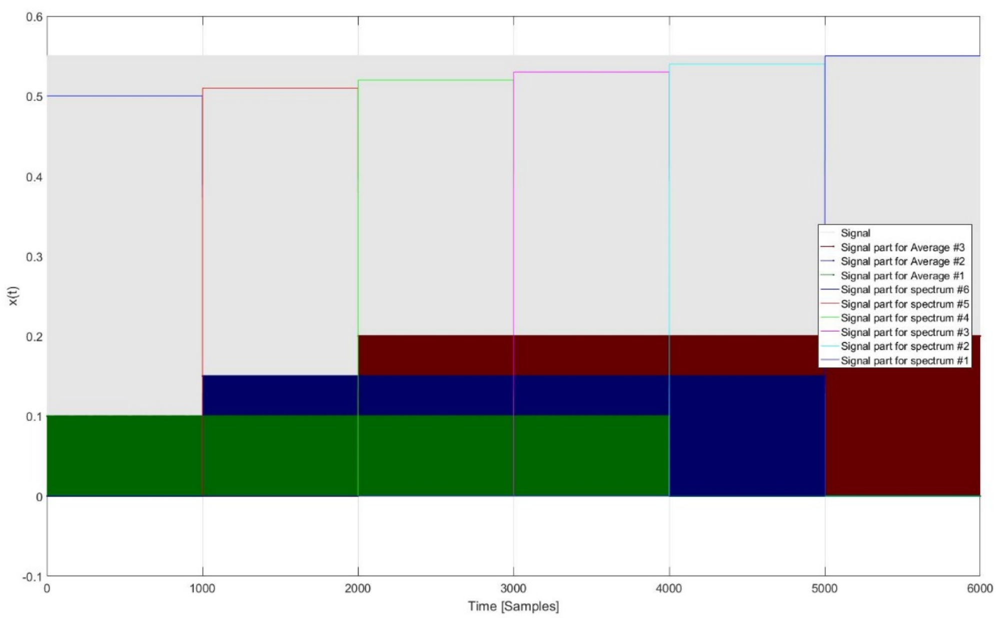


Fig. 8.84: Decomposition of the time signal for a Periodogram with an average of 4 spectra and 0 % overlapping

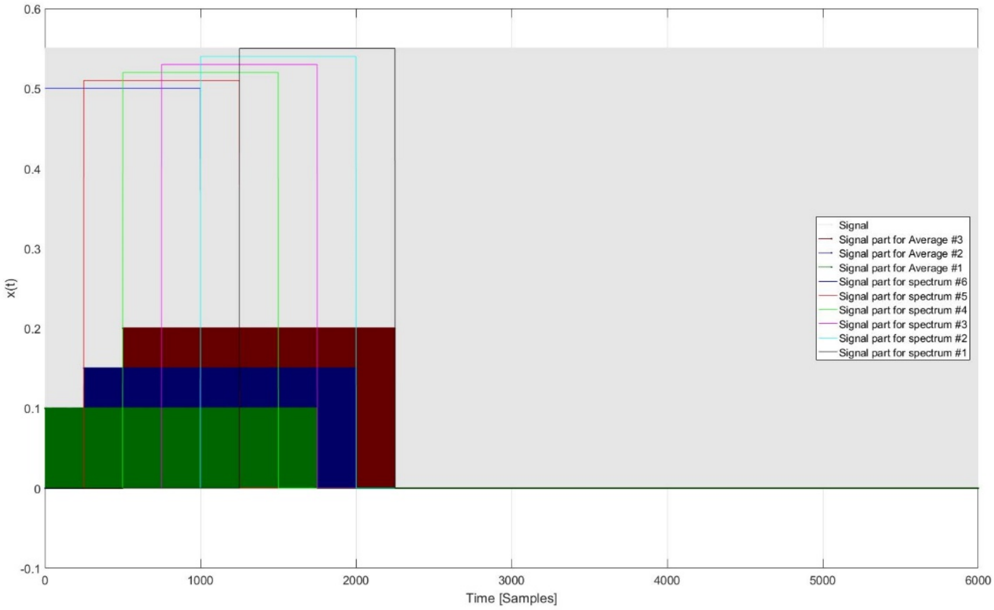


Fig. 8.85: Decomposition of the time signal for a Periodogram with an average of 4 spectra and 75 % overlapping

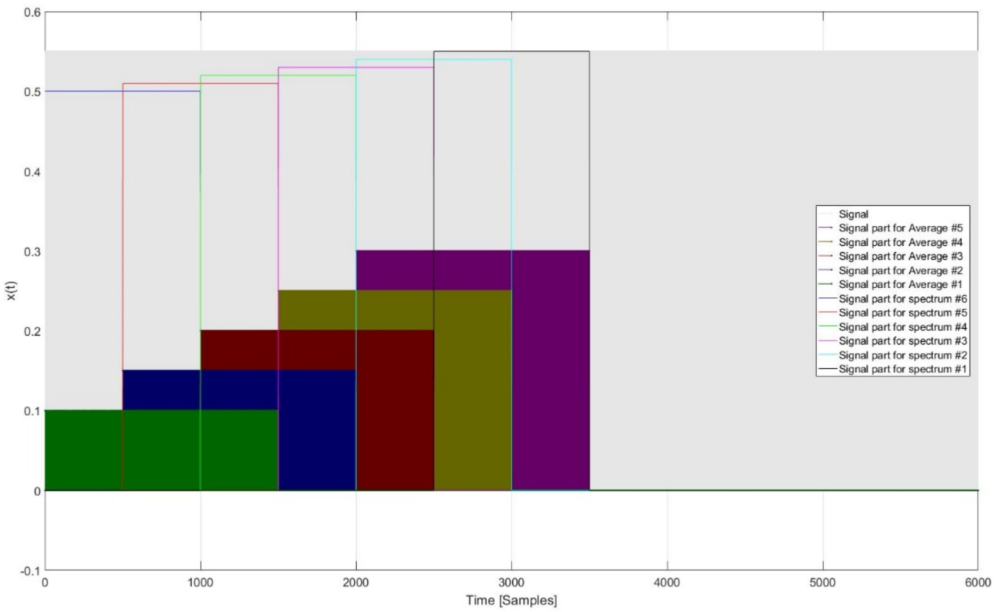


Fig. 8.86: Decomposition of the time signal for a Periodogram with an average of 2 spectra and 50 % overlapping